

Parabolic partial differential equations (1D and 2D)

1. Solve the heat equation problem using the θ method

$$\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2} + 1, \quad t > 0, x \in [0,1]$$

$$t = 0 : T(x, 0) = 0;$$

$$t > 0 : T(0, t) = T(1, t) = 0;$$

2. Solve the 2D heat equation problem

$$\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + 1, \quad t > 0, (x, y) \in D = [0,1] \times [0,1]$$

$$t = 0 : T(x, 0) = 0;$$

$$t > 0 : T|_{\partial D} = 0;$$