Subiect 1

1. Rezolvaţi

$$y' = \frac{1}{t^2} - \frac{y}{t} - y^2, \quad 1 \le t \le 2, \quad y(1) = -1,$$

folosind metoda dezvoltării in serie Taylor de ordin doi pentru un pas h = 0.05. Comparaţi cu soluţia analitica $y(t) = -1/t$ si cu metoda lui Euler explicită.

2.

An article in the 1993 volume of the *Transactions of the American Fisheries Society* reports the results of a study to investigate the mercury contamination in largemouth bass. A sample of fish was selected from 53 Florida lakes and mercury concentration in the muscle tissue was measured (ppm). The mercury concentration values are

1.230	0.490	0.490	1.080	0.590	0.280	0.180	0.100	0.940
1.330	0.190	1.160	0.980	0.340	0.340	0.190	0.210	0.400
0.040	0.830	0.050	0.630	0.340	0.750	0.040	0.860	0.430
0.044	0.810	0.150	0.560	0.840	0.870	0.490	0.520	0.250
1.200	0.710	0.190	0.410	0.500	0.560	1.100	0.650	0.270
0.270	0.500	0.770	0.730	0.340	0.170	0.160	0.270	

We want to find an approximate 95% CI on μ.

Nota: CI = confidence interval, μ - media

Subiect 2

1. Evoluția unei epidemii de gripă într-o populație de N indivizi este modelată de sistemul de ecuații diferențiale

$$\frac{\mathrm{d}x}{\mathrm{d}t} = -\beta xy + \gamma,$$
$$\frac{\mathrm{d}y}{\mathrm{d}t} = \beta xy - \alpha y$$
$$\frac{\mathrm{d}z}{\mathrm{d}t} = \alpha y - \gamma,$$

unde x este numărul de indivizi susceptibili de a face infecția, y este numărul de infectați, iar z este numărul de imuni, care include și numărul de bolnavi refăcuți după boală, la momentul t. Parametrii α, β, γ sunt ratele de recuperare, transmisie şi respectiv recontaminare (replenishment) (pe zi). Se presupune că populația este fixă, astfel că noile naşteri sunt compensate de morți.

Utilizați metoda Runge-Kutta standard pentru a rezolva ecuațiile cu condițiile inițiale $x(0) = 980$, $y(0) = 20$, z(0)=0, dându-se parametrii $\alpha = 0.05$, $\beta = 0.0002$, $\gamma = 0$. Simularea se va termina când $y(t) > 0.9N$. Determinați aproximativ numărul maxim de persoane infectate și momentul când apare.

2.

A civil engineer is analyzing the compressive strength of concrete. Compressive strength is normally distributed with $\sigma^2 = 1000(\text{psi})^2$. A random sample of 12 specimens has a mean compressive strength of $\overline{x} = 3250$ psi.

(a) Construct a 95% two-sided confidence interval on mean compressive strength.

(b) Construct a 99% two-sided confidence interval on mean compressive strength. Compare the width of this confidence interval with the width of the one found in part (a).

Subiect 3

1. Ecuaţia atractorului Lorenz

$$\begin{aligned}\frac{\mathrm{d}x}{\mathrm{d}t} &= -ax + ay,\\ \frac{\mathrm{d}y}{\mathrm{d}t} &= bx - y - xz,\\ \frac{\mathrm{d}z}{\mathrm{d}t} &= -cz + xy\end{aligned}$$

are soluţii haotice care sunt sensibil dependente de condiţiile iniţiale. Rezolvaţi numeric pentru $a = 5$, $b = 15$, $c = 1$ cu condiţiile iniţiale

$$x(0) = 2, \quad y(0) = 6, \quad z(0) = 4, \quad t \in [0, 20],$$

Folositi metoda Runge-Kutta de trei optimi si verificati rezultatele pentru diverse valori ale pasului: 0.1, 0.05, 0.01, etc.

2.

The wall thickness of 25 glass 2-liter bottles was measured by a quality-control engineer. The sample mean was $\bar{x} = 4.05$ millimeters, and the sample standard deviation was $s = 0.08$ millimeter. Find a 95% lower confidence bound for mean wall thickness. Interpret the interval you have obtained.

Subiect 4

1. Rezolvati problema de mai jos folosind metoda Runge-Kutta-Gill

The irreversible chemical reaction in which two molecules of solid potassium dichromate ($K_2Cr_2O_7$), two molecules of water (H_2O), and three atoms of solid sulfur (S) combine to yield three molecules of the gas sulfur dioxide (SO_2), four molecules of solid potassium hydroxide (KOH), and two molecules of solid chromic oxide (Cr_2O_3) can be represented symbolically by the *stoichiometric equation*:

$$2K_2Cr_2O_7 + 2H_2O + 3S \longrightarrow 4KOH + 2Cr_2O_3 + 3SO_2.$$

If n_1 molecules of $K_2Cr_2O_7$, n_2 molecules of H_2O, and n_3 molecules of S are originally available, the following differential equation describes the amount $x(t)$ of KOH after time t:

$$\frac{dx}{dt} = k\left(n_1 - \frac{x}{2}\right)^2\left(n_2 - \frac{x}{2}\right)^2\left(n_3 - \frac{3x}{4}\right)^3,$$

where k is the velocity constant of the reaction. If $k = 6.22 \times 10^{-19}$, $n_1 = n_2 = 2 \times 10^3$, and $n_3 = 3 \times 10^3$, how many units of potassium hydroxide will have been formed after 0.2 s?

2.

(CD WRITER AND BATTERY LIFE). CD writing is energy consuming, therefore, it affects the battery lifetime on laptops. To estimate the effect of CD writing, 30 users are asked to work on their laptops until the "low battery" sign comes on.

Eighteen users without a CD writer worked an average of 5.3 hours with a standard deviation of 1.4 hours. The other twelve, who used their CD writer, worked an average of 4.8 hours with a standard deviation of 1.6 hours. Assuming Normal distributions with equal population variances ($\sigma_X^2 = \sigma_Y^2$), construct a 95% confidence interval for the battery life reduction caused by CD writing.

Subiect 5

1. Cometa Halley şi-a atins ultima dată periheliul (apropierea maximă de soare) la 9 februarie 1986. Poziţia şi componentele vitezei în acel moment erau

$$\begin{aligned}(x, y, z) &= (0.325514, -0.459460, 0.166229)\\ \left(\frac{\mathrm{d}x}{\mathrm{d}t}, \frac{\mathrm{d}y}{\mathrm{d}t}, \frac{\mathrm{d}z}{\mathrm{d}t}\right) &= (-9.096111, -6.916686, -1.305721).\end{aligned}$$

Poziţia este măsurată în unităţi astronomice (distanţa medie de la pământ la soare), iar timpul în ani. Ecuaţiile mişcării sunt

$$\begin{aligned}\frac{\mathrm{d}^2x}{\mathrm{d}t^2} &= -\frac{\mu x}{r^3},\\ \frac{\mathrm{d}^2y}{\mathrm{d}t^2} &= -\frac{\mu y}{r^3},\\ \frac{\mathrm{d}^2z}{\mathrm{d}t^2} &= -\frac{\mu z}{r^3},\end{aligned}$$

unde $r = \sqrt{x^2 + y^2 + z^2}$, $\mu = 4\pi^2$, iar perturbaţiile planetare au fost neglijate. Rezolvaţi aceste ecuaţii numeric pentru a determina aproximativ momentul următorului periheliu.

Nota: Folositi o metoda de ordinul doi.

2.

An automatic filling machine is used to fill bottles with liquid detergent. A random sample of 20 bottles results in a sample variance of fill volume of $s^2 = 0.0153$ (fluid ounces)^2. If the variance of fill volume exceeds 0.01 (fluid ounces)^2, an unacceptable proportion of bottles will be underfilled or overfilled. Is there evidence in the sample data to suggest that the manufacturer has a problem with underfilled or overfilled bottles? Use $\alpha = 0.05$, and assume that fill volume has a normal distribution.

Subiect 6

1. The solution of the problem

$$y'' + \frac{1}{x}y' + \left(1 - \frac{1}{x^2}\right)y \qquad y(0) = 0 \qquad y'(0) = 1$$

is the Bessel function $J_1(x)$. Use numerical integration to compute $J_1(5)$ and compare the result with $-0.327\,579$, the value listed in mathematical tables. *Hint*: to avoid singularity at $x = 0$, start the integration at $x = 10^{-12}$.

Nota: Folositi Metoda Runge-Kuta de trei optimi.

2.

Extracts of St. John's Wort are widely used to treat depression. An article in the April 18, 2001 issue of the *Journal of the American Medical Association* ("Effectiveness of St. John's Wort on Major Depression: A Randomized Controlled Trial") compared the efficacy of a standard extract of St. John's Wort with a placebo in 200 outpatients diagnosed with major depression. Patients were randomly assigned to two groups; one group received the St. John's Wort, and the other received the placebo. After eight weeks, 19 of the placebo-treated patients showed improvement, whereas 27 of those treated with St. John's Wort improved. Is there any reason to believe that St. John's Wort is effective in treating major depression? Use $\alpha = 0.05$.

Subiect 7

1. Rezolvati ecuatia

$$f'''+\frac{1}{2}ff''+e^{-\eta}=0$$

cu condiţiile

$$f(0)=0, \quad f'(0)=0, \; f'(\eta_\infty)=1$$

unde f = f(η), $\eta \in [0,\eta_\infty]$,iar η_∞= 10.

2.

A product developer is interested in reducing the drying time of a primer paint. Two formulations of the paint are tested; formulation 1 is the standard chemistry, and formulation 2 has a new drying ingredient that should reduce the drying time. From experience, it is known that the standard deviation of drying time is 8 minutes, and this inherent variability should be unaffected by the addition of the new ingredient. Ten specimens are painted with formulation 1, and another 10 specimens are painted with formulation 2; the 20 specimens are painted in random order. The two sample average drying times are $\bar{x}_1 = 121$ minutes and $\bar{x}_2 = 112$ minutes, respectively. What conclusions can the product developer draw about the effectiveness of the new ingredient, using $\alpha = 0.05$?

Subiect 8

1. Rezolvati sistemul pentru $n = 2$ si $\lambda = 1$

$$(f')^n = \theta$$

$$\theta'' + \frac{1}{2}\frac{n+\lambda}{n} f\theta' - \lambda f'\theta = 0$$

$$f(0) = 0,\ \ \theta(0) = 1,\ \theta(0) = 0$$

unde f = f(η) si $\theta = \theta(\eta)$, $\eta \in [0, \eta_\infty]$, iar $\eta_\infty = 10$.

2.

Two catalysts are being analyzed to determine how they affect the mean yield of a chemical process. Specifically, catalyst 1 is currently in use, but catalyst 2 is acceptable. Since catalyst 2 is cheaper, it should be adopted, providing it does not change the process yield. A test is run in the pilot plant and results in the data shown in Table 10-1. Is there any difference between the mean yields? Use $\alpha = 0.05$, and assume equal variances.

Table 10-1 Catalyst Yield Data, Exar

Observation Number	Catalyst 1	Catalyst 2
1	91.50	89.19
2	94.18	90.95
3	92.18	90.46
4	95.39	93.21
5	91.79	97.19
6	89.07	97.04
7	94.72	91.07
8	89.21	92.75

Subiect 9

1. Rezolvati ecuatia pentru $\alpha = 0.154$ si $\infty = 4$

$$\frac{d^2f}{d\eta^2} + \frac{2\eta}{\{1-[\alpha/(1-\alpha)]f\}^{1/2}} \frac{df}{d\eta} = 0$$

$$f(0) = 0, \qquad f(\infty) = -1$$

2.

Arsenic concentration in public drinking water supplies is a potential health risk. An article in the *Arizona Republic* (Sunday, May 27, 2001) reported drinking water arsenic concentrations in parts per billion (ppb) for 10 methropolitan Phoenix communities and 10 communities in rural Arizona. The data follow:

Metro Phoenix	Rural Arizona
Phoenix, 3	Rimrock, 48
Chandler, 7	Goodyear, 44
Gilbert, 25	New River, 40
Glendale, 10	Apachie Junction, 38
Mesa, 15	Buckeye, 33
Paradise Valley, 6	Nogales, 21
Peoria, 12	Black Canyon City, 20
Scottsdale, 25	Sedona, 12
Tempe, 15	Payson, 1
Sun City, 7	Casa Grande, 18

We wish to determine it there is any difference in mean arsenic concentrations between metropolitan Phoenix communities and communities in rural Arizona.

Subiect 10

1. Rezolvati ecuatia pentru $x_{A0} = 0.25$ (vezi Na, 1979)

$$\frac{d^2x_A}{d\xi^2} + \frac{1}{2(1-\frac{1}{2}x_A)}\left(\frac{dx_A}{d\xi}\right)^2 = 0$$

$$\xi = 0: \quad x_A = x_{A0}; \qquad \xi = 1: \quad x_A = 0$$

2.

An article in the *Journal of Strain Analysis* (1983, Vol. 18, No. 2) compares several methods for predicting the shear strength for steel plate girders. Data for two of these methods, the Karlsruhe and Lehigh procedures, when applied to nine specific girders, are shown in Table 10-2. We wish to determine whether there is any difference (on the average) between the two methods.

Table 10-2 Strength Predictions for Nine Steel Plate Girders (Predicted Load/Observed Load)

Girder	Karlsruhe Method	Lehigh Method	Difference d_j
S1/1	1.186	1.061	0.119
S2/1	1.151	0.992	0.159
S3/1	1.322	1.063	0.259
S4/1	1.339	1.062	0.277
S5/1	1.200	1.065	0.138
S2/1	1.402	1.178	0.224
S2/2	1.365	1.037	0.328
S2/3	1.537	1.086	0.451
S2/4	1.559	1.052	0.507

Subiect 11

1. Rezolvati ecuatia pentru $\tau_f = 0.5$ si $\theta_f = 0.2618$ (vezi Na, 1979)

$$\frac{d^2\theta}{d\tau^2} + \sin\theta = 0$$

$$\tau = 0: \quad \theta = 0; \qquad \tau = \tau_f: \quad \theta = \theta_f$$

2.

Use MATLAB to generate eight points from the function

$$f(t) = \sin^2 t$$

from $t = 0$ to 2π. Fit this data using **(a)** cubic spline with not-a-knot end conditions, **(b)** cubic spline with derivative end conditions equal to the exact values calculated with differentiation, and **(c)** piecewise cubic hermite interpolation. Develop plots of each fit as well as plots of the absolute error (E_t = approximation − true) for each.

Subiect 12

1. Rezolvati ecuatia

$$y'' = 4(y - x), \quad 0 \le x \le 1, \quad y(0) = 0, \quad y(1) = 2,$$

Comparati cu solutia analitica

$$y(x) = e^2(e^4 - 1)^{-1}(e^{2x} - e^{-2x}) + x$$

2.

Dynamic viscosity of water $\mu(10^{-3}\ \mathrm{N \cdot s/m^2})$ is related to temperature $T(°C)$ in the following manner:

T	0	5	10	20	30	40
μ	1.787	1.519	1.307	1.002	0.7975	0.6529

(a) Plot this data.
(b) Use linear interpolation to predict μ at $T = 7.5$ °C.
(c) Use polynomial regression to fit a parabola to the data in order to make the same prediction.

Subiect 13

1. Rezolvati ecuatia

$$y'' = 2y' - y + xe^x - x, \quad 0 \le x \le 2, \; y(0) = 0, \; y(2) = -4;$$

Comparati cu solutia analitica

$$y(x) = \tfrac{1}{6}x^3e^x - \tfrac{5}{3}xe^x + 2e^x - x - 2.$$

2.

The following model is used to represent the effect of solar radiation on the photosynthesis rate of aquatic plants:

$$P = P_m \frac{I}{I_{sat}} e^{-\frac{I}{I_{sat}}+1}$$

where P = the photosynthesis rate (mg $m^{-3}d^{-1}$), P_m = the maximum photosynthesis rate (mg $m^{-3}d^{-1}$), I = solar radiation (μE $m^{-2}s^{-1}$), and I_{sat} = optimal solar radiation (μE $m^{-2}s^{-1}$). Use nonlinear regression to evaluate P_m and I_{sat} based on the following data:

I	50	80	130	200	250	350	450	550	700
P	99	177	202	248	229	219	173	142	72

Subiect 14

1. Rezolvati ecuatia

$$y'' = \frac{1}{8}(32 + 2x^3 - yy'), \quad 1 \le x \le 3, \quad y(1) = 17, \quad y(3) = \frac{43}{3},$$

Comparati cu solutia analitica

$$y(x) = x^2 + 16/x.$$

2.

An investigator has reported the data tabulated below for an experiment to determine the growth rate of bacteria k (per d) as a function of oxygen concentration c (mg/L). It is known that such data can be modeled by the following equation:

$$k = \frac{k_{max} c^2}{c_s + c^2}$$

where c_s and k_{max} are parameters. Use a transformation to linearize this equation. Then use linear regression to estimate c_s and k_{max} and predict the growth rate at $c = 2$ mg/L.

c	0.5	0.8	1.5	2.5	4
k	1.1	2.4	5.3	7.6	8.9

Subiect 15

1. Rezolvati ecuatia

$$y'' = 2y^3, \quad -1 \le x \le 0, \quad y(-1) = \frac{1}{2}, \quad y(0) = \frac{1}{3}.$$

Comparati cu solutia analitica

$$y(x) = 1/(x+3).$$

2.

The following data was taken from a stirred tank reactor for the reaction $A \to B$. Use the data to determine the best possible estimates for k_{01} and E_1 for the following kinetic model:

$$-\frac{dA}{dt} = k_{01} e^{-E_1/RT} A$$

where R is the gas constant and equals 0.00198 kcal/mol/K.

$-dA/dt$ (moles/L/s)	460	960	2485	1600	1245
A (moles/L)	200	150	50	20	10
T (K)	280	320	450	500	550

Subiect 16

1. Rezolvati ecuatia

$$y'' = -\frac{2}{x}y' + \frac{2}{x^2}y + \frac{\sin(\ln x)}{x^2}, \quad 1 \le x \le 2, \quad y(1) = 1, \quad y(2) = 2.$$

Comparati cu solutia analitica

$$y = c_1 x + \frac{c_2}{x^2} - \frac{3}{10}\sin(\ln x) - \frac{1}{10}\cos(\ln x),$$

$$c_2 = \frac{1}{70}[8 - 12\sin(\ln 2) - 4\cos(\ln 2)] \approx -0.03920701320$$

$$c_1 = \frac{11}{10} - c_2 \approx 1.1392070132.$$

2.

An experiment is performed to determine the % elongation of electrical conducting material as a function of temperature. The resulting data is listed below. Predict the % elongation for a temperature of 400 °C.

Temperature, °C	200	250	300	375	425	475	600
% Elongation	7.5	8.6	8.7	10	11.3	12.7	15.3

Subiect 17

1.Rezolvati ecuatia

The lead example of this chapter concerned the deflection of a beam with supported ends subject to uniform loading. The boundary-value problem governing this physical situation is

$$\frac{d^2w}{dx^2} = \frac{S}{EI}w + \frac{qx}{2EI}(x - l), \quad 0 < x < l,$$

with boundary conditions $w(0) = 0$ and $w(l) = 0$.

Suppose the beam is a W10-type steel I-beam with the following characteristics: length $l = 120$ in., intensity of uniform load $q = 100$ lb/ft, modulus of elasticity $E = 3.0 \times 10^7$ lb/in.^{2}, stress at ends $S = 1000$ lb, and central moment of inertia $I = 625$ in.^{4}.

2.

Andrade's equation has been proposed as a model of the effect of temperature on viscosity:

$$\mu = De^{B/T_a}$$

where μ = dynamic viscosity of water (10^{-3} N·s/m^2), T_a = absolute temperature (K), and D and B are parameters. Fit this model to the following data for water:

T	0	5	10	20	30	40
μ	1.787	1.519	1.307	1.002	0.7975	0.6529

Subiect 18

1.

For values of x in the interval $[-1, 3]$ and t in $[0, 2.4]$, solve the one-way wave equation

$$u_t + u_x = 0,$$

with the initial data

$$u(0,x) = \begin{cases} \cos^2 \pi x & \text{if } |x| \le \frac{1}{2}, \\ 0 & \text{otherwise,} \end{cases}$$

and the boundary data $u(t, -1) = 0$.

2.

The following data was collected for the steady flow of water in a concrete circular pipe:

Experiment	Diameter, m	Slope, m/m	Flow, m^3/s
1	0.3	0.001	0.04
2	0.6	0.001	0.24
3	0.9	0.001	0.69
4	0.3	0.01	0.13
5	0.6	0.01	0.82
6	0.9	0.01	2.38
7	0.3	0.05	0.31
8	0.6	0.05	1.95
9	0.9	0.05	5.66

Use multiple linear regression to fit the following model to this data:

$$Q = \alpha_0 D^{\alpha_1} S^{\alpha_2}$$

where Q = flow, D = diameter, and S = slope.

-

Subiect 19

1.

Solve the initial boundary value problem for $u_t = u_{xx}$ on $-1 \le x \le 1$ for $0 \le t \le 0.5$ with initial data given by

$$u_0(x) = \begin{cases} 1 - |x| & \text{for } |x| < \frac{1}{2}, \\ \frac{1}{4} & \text{for } |x| = \frac{1}{2}, \\ 0 & \text{for } |x| > \frac{1}{2}. \end{cases}$$

Use the boundary conditions

$$u(t, -1) = u^*(t, -1) \quad \text{and} \quad u_x(t, 1) = 0,$$

where $u^*(t, x)$ is the exact solution given by

$$u^*(t, x) = \frac{3}{8} + \sum_{\ell=0}^{\infty} \left(\frac{(-1)^\ell}{\pi(2\ell + 1)} + \frac{2}{\pi^2(2\ell + 1)^2} \right) \cos \pi(2\ell + 1)x \, e^{-\pi^2(2\ell+1)^2 t}$$
$$+ \sum_{m=0}^{\infty} \frac{\cos 2\pi(2m + 1)x}{\pi^2(2m + 1)^2} e^{-4\pi^2(2m+1)^2 t}.$$

2.

Environmental scientists and engineers dealing with the impacts of acid rain must determine the value of the ion product of water K_w as a function of temperature. Scientists have suggested the following equation to model this relationship:

$$-\log_{10} K_w = \frac{a}{T_a} + b \log_{10} T_a + cT_a + d$$

where T_a = absolute temperature (K), and a, b, c, and d are parameters. Employ the following data and regression to estimate the parameters:

***T* (K)**	K_w
0	1.164×10^{-15}
10	2.950×10^{-15}
20	6.846×10^{-15}
30	1.467×10^{-14}
40	2.929×10^{-14}

Subiect 20

1. Solve

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \qquad 1 < x < 2, \quad 0 < y < 1;$$

$$u(x, 0) = 2 \ln x, \qquad u(x, 1) = \ln(x^2 + 1), \quad 1 \leq x \leq 2;$$

$$u(1, y) = \ln(y^2 + 1), \quad u(2, y) = \ln(y^2 + 4), \quad 0 \leq y \leq 1.$$

and compare the results to the actual solution $u(x, y) = \ln(x^2 + y^2)$.

2.

The distance required to stop an automobile consists of both thinking and braking components, each of which is a function of its speed. The following experimental data was collected to quantify this relationship. Develop best-fit equations for both the thinking and braking components. Use these equations to estimate the total stopping distance for a car traveling at 110 km/h.

Speed, km/h	30	45	60	75	90	120
Thinking, m	5.6	8.5	11.1	14.5	16.7	22.4
Braking, m	5.0	12.3	21.0	32.9	47.6	84.7

Subiect 21

1. Solve

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \qquad 0 < x < 1, \quad 0 < y < 1;$$

$$u(x,0) = 0, \quad u(x,1) = x, \qquad 0 \le x \le 1;$$

$$u(0,y) = 0, \quad u(1,y) = y, \qquad 0 \le y \le 1.$$

Use $h = k = 0.2$, and compare the results to the actual solution $u(x, y) = xy$.

2.

The solar radiation for Tucson, Arizona, has been tabulated as

Time, mo	J	F	M	A	M	J	J	A	S	O	N	D
Radiation, W/m²	144	188	245	311	351	359	308	287	260	211	159	131

Assuming each month is 30 days long, fit a sinusoid to this data. Use the resulting equation to predict the radiation in mid-August.

Subiect 22

1. Solve

$$\frac{\partial u}{\partial t} - \frac{\partial^2 u}{\partial x^2} = 0, \quad 0 < x < 2,\ 0 < t;$$

$$u(0, t) = u(2, t) = 0, \quad 0 < t,$$

$$u(x, 0) = \sin 2\pi x, \quad 0 \leq x \leq 2.$$

Use $h = 0.4$ and $k = 0.1$, and compare your results at $t = 0.5$ to the actual solution $u(x, t) = e^{-4\pi^2 t} \sin 2\pi x$. Then use $h = 0.4$ and $k = 0.05$, and compare the answers.

2. Bessel functions often arise in advanced engineering analyses such as the study of electric fields. Here are some selected values for the zero-order Bessel function of the first kind

x	1.8	2.0	2.2	2.4	2.6
$J_1(x)$	0.5815	0.5767	0.5560	0.5202	0.4708

Estimate $J_1(2.1)$ using third-, and fourth-order interpolating polynomials. Determine the percent relative error for each case based on the true value, which can be determined with MATLAB's built-in function `besselj`.

Subiect 23

1. Solve

$$\frac{\partial u}{\partial t} - \frac{\partial^2 u}{\partial x^2} = 0, \quad 0 < x < \pi, \ 0 < t;$$

$$u(0, t) = u(\pi, t) = 0, \quad 0 < t,$$

$$u(x, 0) = \sin x, \quad 0 \le x \le \pi.$$

Use $h = \pi/10$ and $k = 0.05$, and compare your results at $t = 0.5$ to the actual solution $u(x, t) = e^{-t} \sin x$.

2. The following is the built-in humps function that MATLAB uses to demonstrate some of its numerical capabilities:

$$f(x) = \frac{1}{(x - 0.3)^2 + 0.01} + \frac{1}{(x - 0.9)^2 + 0.04} - 6$$

The humps function exhibits both flat and steep regions over a relatively short x range. Here are some values that have been generated at intervals of 0.1 over the range from $x = 0$ to 1:

x	0	0.1	0.2	0.3	0.4	0.5
$f(x)$	5.176	15.471	45.887	96.500	47.448	19.000
x	0.6	0.7	0.8	0.9	1	
$f(x)$	11.692	12.382	17.846	21.703	16.000	

Fit this data with a **(a)** cubic spline with not-a-knot end conditions and **(b)** piecewise cubic Hermite interpolation. In both cases, create a plot comparing the fit with the exact humps function.

Subiect 24

1. Solve

$$\frac{\partial u}{\partial t} - \frac{1}{\pi^2}\frac{\partial^2 u}{\partial x^2} = 0, \quad 0 < x < 1,\ 0 < t;$$

$$u(0,t) = u(1,t) = 0, \quad 0 < t,$$

$$u(x,0) = \cos \pi \left(x - \tfrac{1}{2}\right), \quad 0 \le x \le 1.$$

Use $h = 0.1$ and $k = 0.04$, and compare your results at $t = 0.4$ to the actual solution $u(x,t) = e^{-t} \cos \pi (x - \frac{1}{2})$.

2. Scrieti un program matlab de interpolare a unei functii f cu functii spline cubice pe un interval [a, b] considerand conditia de spline de Boor in punctual a ($p_1(x) \equiv p_2(x)$) si spline natural in punctual b ($s''(f;b) = 0$). Aplicatie: aproximati sin(x) pe intervalul [0, 2π].

Subiect 25

1. Solve

$$\begin{aligned} u_t &= u_{xx} + xtu_x + xtu, \\ u(0,t) &= e^t, \quad 0 \le t \le 1, \\ u_x(1,t) &= -u(1,t), \\ u(x,0) &= e^{-x}, \quad 0 \le x \le 1, \end{aligned}$$

which has the exact solution $u(x,t) = e^{t-x}$

2. Scrieti un program matlab de interpolare a unei functii f cu functii spline cubice pe un interval [a, b] considerand conditia de spline complet in punctual a ($m_1 = f'(a)$) si spline cu derivate secunde in punctual b ($s_3''(f;b) = f''(b)$). Aplicatie: aproximati sin(x) pe intervalul [0, 2π].

Subiect 26

1. Solve

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = -(\cos(x+y) + \cos(x-y)), \qquad 0 < x < \pi, \quad 0 < y < \frac{\pi}{2};$$

$$u(0, y) = \cos y, \quad u(\pi, y) = -\cos y, \qquad 0 \le y \le \frac{\pi}{2},$$

$$u(x, 0) = \cos x, \quad u\left(x, \frac{\pi}{2}\right) = 0, \qquad 0 \le x \le \pi.$$

Use $h = \pi/5$ and $k = \pi/10$, and compare the results to the actual solution $u(x, y) = \cos x \cos y$.

2. Scrieti un program matlab de interpolare a unei functii f cu functii spline liniare. Aproximati sin(x) pe intervalul [0, 2π] folosind functii spline liniare.

Subiect 27

1.

The temperature $u(x, t)$ of a long, thin rod of constant cross section and homogeneous conducting material is governed by the one-dimensional heat equation. If heat is generated in the material, for example, by resistance to current or nuclear reaction, the heat equation becomes

$$\frac{\partial^2 u}{\partial x^2} + \frac{Kr}{\rho C} = K\frac{\partial u}{\partial t}, \quad 0 < x < l, \quad 0 < t,$$

where l is the length, ρ is the density, C is the specific heat, and K is the thermal diffusivity of the rod. The function $r = r(x, t, u)$ represents the heat generated per unit volume. Suppose that

$$l = 1.5 \text{ cm}, \qquad K = 1.04 \text{ cal/cm} \cdot \text{deg} \cdot \text{s},$$
$$\rho = 10.6 \text{ g/cm}^3, \qquad C = 0.056 \text{ cal/g} \cdot \text{deg},$$

and

$$r(x, t, u) = 5.0 \text{ cal/cm}^3 \cdot \text{s}.$$

If the ends of the rod are kept at 0°C, then

$$u(0, t) = u(l, t) = 0, \quad t > 0.$$

Suppose the initial temperature distribution is given by

$$u(x, 0) = \sin\frac{\pi x}{l}, \quad 0 \le x \le l.$$

Aflati distributia temperaturii la diferite momente de timp folosind o metoda implicita. Reprezentati grafic.

2. Cate noduri echidistante in intervalul $[0, \pi]$ sunt necesare pentru a aproxima $\sin(\pi/5)$ cu eroare err = 1e-3 folosind interpolarea Lagrange?

Subiect 28

1.

Sagar and Payne [SP] analyze the stress-strain relationships and material properties of a cylinder alternately subjected to heating and cooling and consider the equation

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} = \frac{1}{4K}\frac{\partial T}{\partial t}, \quad \frac{1}{2} < r < 1, \ 0 < T,$$

where $T = T(r, t)$ is the temperature, r is the radial distance from the center of the cylinder, t is time, and K is a diffusivity coefficient.

a. Find approximations to $T(r, 10)$ for a cylinder with outside radius 1, given the initial and boundary conditions:

$$T(1, t) = 100 + 40t, \quad 0 \le t \le 10;$$

$$T\left(\frac{1}{2}, t\right) = t, \quad 0 \le t \le 10;$$

$$T(r, 0) = 200(r - 0.5), \quad 0.5 \le r \le 1.$$

2. In urmatorul tabel este data variatia densitatii apei cu temperatura:

T(^{o}C)	0	4	10	15	20	22	25	30	40	60	80	100
ρ (kg/m^3)	999.84	999.97	999.7	999.1	998.21	997.77	997.05	995.65	992.2	983.2	971.8	958.4

O lege care descrie aceasta variatie este aproximatia lui Boussinesq: $\rho = \rho_0(1 - \beta(T - T_0))$ unde temperatura este exprimata in grade Kelvin (T(^{o}C)= T(K) + 273.15), iar ρ_0 si T_0 reprezinta valorile la 0 ^{o}C. Aflati coeficientul β.

Subiect 29

1. A reactor is thermally stratified as in the following table:

Depth, m	0	0.5	1	1.5	2	2.5	3
Temperature, °C	70	70	55	22	13	10	10

Based on these temperatures, the tank can be idealized as two zones separated by a strong temperature gradient or *thermocline*. The depth of the thermocline can be defined as the inflection point of the temperature-depth curve—that is, the point at which $d^2T/dz^2 = 0$. At this depth, the heat flux from the surface to the bottom layer can be computed with Fourier's law:

$$J = -k\frac{dT}{dz}$$

Use a clamped cubic spline fit with zero end derivatives to determine the thermocline depth. If $k = 0.01$ cal/(s · cm · °C) compute the flux across this interface.

2.

Oxide layers on semiconductor wafers are etched in a mixture of gases to achieve the proper thickness. The variability in the thickness of these oxide layers is a critical characteristic of the wafer, and low variability is desirable for subsequent processing steps. Two different mixtures of gases are being studied to determine whether one is superior in reducing the variability of the oxide thickness. Twenty wafers are etched in each gas. The sample standard deviations of oxide thickness are $s_1 = 1.96$ angstroms and $s_2 = 2.13$ angstroms, respectively. Is there any evidence to indicate that either gas is preferable? Use $\alpha = 0.05$.