Steady free convection in a rectangular cavity filled with a porous medium

1. Introduction

Natural convective heat transfer in fluid-saturated porous media has occupied the center stage in many fundamental heat transfer analyses and has received a considerable attention over the last several decades. This interest was estimated due to its wide range of applications in, for example, packed sphere beds, high performance insulation for buildings, chemical catalytic reactors, grain storage and such geophysical problems as frost heave. Porous media are also of interest in relation to the underground spread of pollutants, solar power collectors, and to geothermal energy systems. Literature concerning convective flow in porous media is abundant. Representative studies in this are may be found in the recent books by Nield and Bejan (2006), Ingham and Pop (2005), Vafai (2005), Bejan *et al.* (2004) and Pop and Ingham (2001).

Natural convection in an enclosure in which internal heat generation is present is of prime importance in certain technological applications. Examples are post-accident heat removal in nuclear reactors and geophysical problems associated with the underground storage of nuclear water, among others (Acharya and Goldstein, 1985; Ozoe and Maruo, 1987; Lee and Goldstein, 1988; Fusegi *et al*., 1992; Venkatachalappa and Subbaraya, 1993; Shim and Hyun, 1997; Hossain and Wilson, 2002; Hossain and Rees, 2005).

2. Mathematical model

Consider the steady natural convection flow in a rectangular cavity filled with a fluid-saturated porous medium and an internal heat generation. The geometry and the Cartesian coordinate system are schematically shown in Fig. 1, where the dimensional coordinates x and y are measured along the horizontal bottom wall and normal to it along the left vertical wall, respectively. The height of the cavity is denoted by h and the width by l, respectively. It is assumed that the vertical walls are maintained at a constant temperature T_0, while the horizontal walls are adiabatic. We also bring into account the effect of a uniform heat generation in the flow region. The constant volumetric rate of heat generation is $q_0''' \, [W/m^3]$. It is also assumed that the effect of buoyancy is included through the well-known Boussinesq approximation.. The resulting convective flow is governed by the combined mechanism of the driven buoyant force, and internal heat generation. Under the above assumptions, the conservation equations for mass, Darcy, energy and electric transfer are

$$\nabla \cdot \mathbf{V} = 0 \tag{1}$$

$$\mathbf{V} = \frac{K}{\mu}\left(-\nabla p + \rho\,\mathbf{g}\right) \tag{2}$$

$$\left(\mathbf{V}\cdot\nabla\right)T = \alpha_m \nabla^2 T + \frac{q_0'''}{\rho_0 c_p} \tag{3}$$

$$\rho = \rho_0\left[1 - \beta\left(T - T_0\right)\right] \tag{4}$$

where V is the velocity vector, T is the fluid temperature, p is the pressure, g is the acceleration vector, K is the permeability of the porous medium, α_m is the effective thermal diffusivity, ρ is the density, μ is the dynamic viscosity, β is the coefficient of thermal expansion, c_p is the specific heat at constant pressure, ρ_0 is the reference density.

Eliminating the pressure term in Eq. (2) in the usual way, the governing equations (1) to (3) can be written as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{5}$$

$$\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} = -\frac{gK\beta}{\nu}\frac{\partial T}{\partial x} \tag{6}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha_m\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + \frac{q_0'''}{\rho c_p} \tag{7}$$

and are subjected to the boundary conditions

$$u=0,\quad T=T_0,\quad \text{at}\quad x=0\quad \text{and}\quad x=h$$
$$v=0,\quad \frac{\partial T}{\partial y}=0,\quad \text{at}\quad y=0\quad \text{and}\quad y=h \tag{8}$$

where υ is the kinematic viscosity. Further, we introduce the following non-dimensional variables

$$X=\frac{x}{l},\quad Y=\frac{y}{h},\quad U=\frac{h}{\alpha_m}u,\quad V=\frac{l}{\alpha_m}v,\quad \theta=\frac{T-T_0}{\left(q_0'''l^2/k\right)} \tag{9}$$

where k is the thermal conductivity.

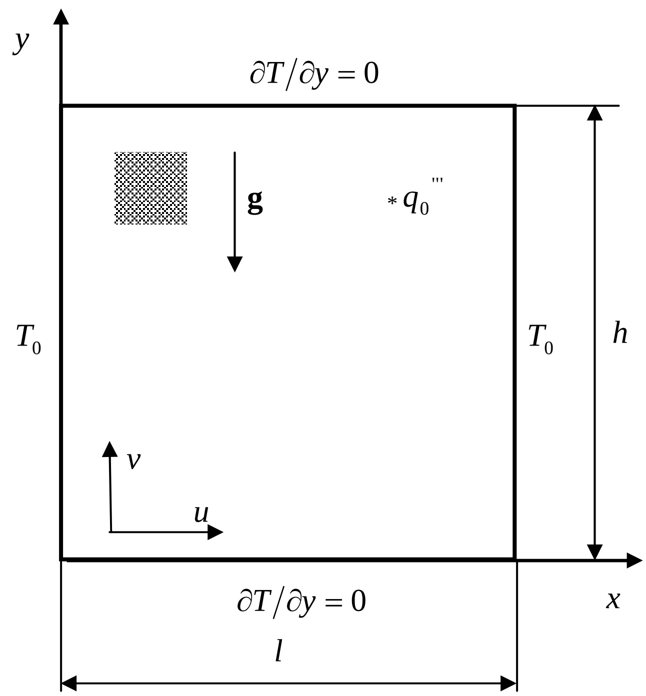


Figure 1.Geometry of the problem and co-ordinate system

Introducing the stream function ψ defined as $U=\partial\psi/\partial Y$ and $V=-\partial\psi/\partial X$, and using (9) in Eqs. (5) - (8), we obtain the following partial differential equations in non-dimensional form:

$$\frac{\partial^2\psi}{\partial X^2}+a^2\frac{\partial^2\psi}{\partial Y^2}=-Ra\frac{\partial\theta}{\partial X} \tag{10}$$

$$\frac{\partial^2\theta}{\partial X^2}+a^2\frac{\partial^2\theta}{\partial Y^2}+1=a\left(\frac{\partial\psi}{\partial Y}\frac{\partial\theta}{\partial X}-\frac{\partial\psi}{\partial X}\frac{\partial\theta}{\partial Y}\right) \tag{11}$$

subject to the boundary conditions

$$U=0,\quad \psi=0,\quad \theta=0,\quad \text{at}\quad X=0\quad \text{and}\quad X=1$$
$$V=0,\quad \psi=0,\quad \frac{\partial\theta}{\partial Y}=0,\quad \text{at}\quad Y=0\quad \text{and}\quad Y=1 \tag{12}$$

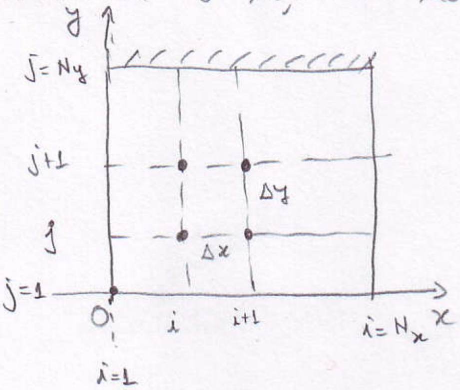
where $a=l/h$ is the aspect ratio of the cavity and *Ra* is the Rayleigh number.

Once we know the numerical values of the temperature function we may obtain the rate of heat flux from each of the vertical walls. The non-dimensional heat transfer rate, q_w, per unit length in the depthwise direction for the left vertical wall is given by

$$Nu_Y=-\left(\frac{\partial\theta}{\partial X}\right)_{X=0},\ Nu=-\int_0^1\left(\frac{\partial\theta}{\partial X}\right)_{X=0}dY \tag{13}$$

Metoda numerică:

Considerăm o rețea de noduri echidistante în direcțiile Ox și Oy cu pașii Δx și Δy.



Aproximăm o funcție de două variabile $f(x,y)$ într-un punct (i,j) astfel:

$$\frac{\partial f}{\partial x} = \frac{f_{i+1,j} - f_{i-1,j}}{2\Delta x} \; ; \; \frac{\partial^2 f}{\partial x^2} = \frac{f_{i+1,j} - 2f_{ij} + f_{i-1,j}}{(\Delta x)^2}$$

$$\frac{\partial f}{\partial y} = \frac{f_{i,j+1} - f_{i,j-1}}{2\Delta y} \; ; \; \frac{\partial^2 f}{\partial y^2} = \frac{f_{i,j+1} - 2f_{ij} + f_{i,j-1}}{(\Delta y)^2}$$

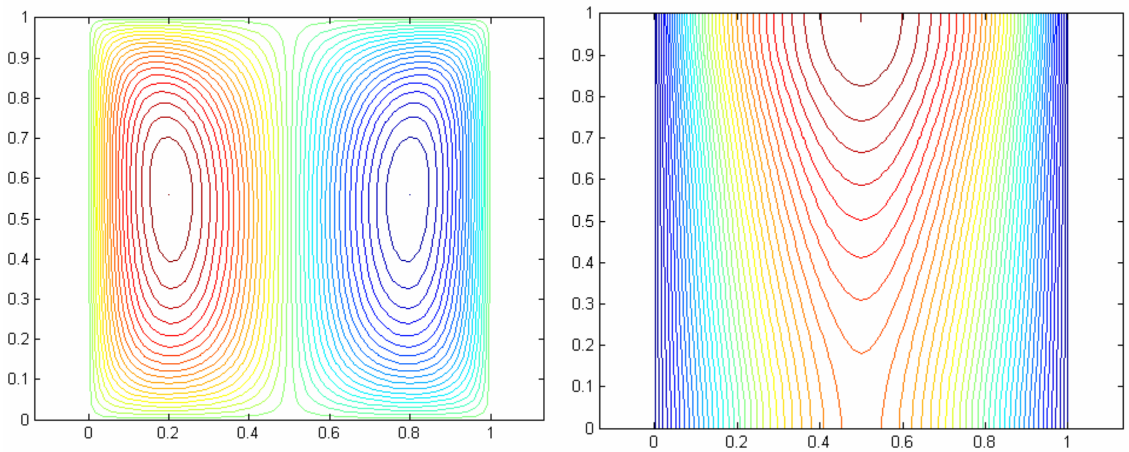
Așadar ecuațiile (10) – (11) devin:

$$i,j = \overline{2,} \begin{cases} \dfrac{\Psi_{i+1,j} - 2\Psi_{i,j} + \Psi_{i-1,j}}{(\Delta x)^2} + a^2 \dfrac{\Psi_{i,j+1} - 2\Psi_{ij} + \Psi_{i,j-1}}{(\Delta y)^2} = -Ra \dfrac{\theta_{i+1,j} - \theta_{i-1,j}}{2\cdot\Delta x} \\[2ex] \dfrac{\theta_{i+1,j} - 2\theta_{ij} + \theta_{i-1,j}}{(\Delta x)^2} + a^2 \dfrac{\theta_{i,j+1} - 2\theta_{ij} + \theta_{i,j-1}}{(\Delta y)^2} + 1 = \\[2ex] \qquad a\left(\dfrac{\Psi_{i,j+1} - \Psi_{i,j-1}}{2\cdot\Delta y} \cdot \dfrac{\theta_{i+1,j} - \theta_{i-1,j}}{2\cdot\Delta x} - \dfrac{\Psi_{i+1,j} - \Psi_{i-1,j}}{2\cdot\Delta x} \cdot \dfrac{\theta_{i,j+1} - \theta_{i,j-1}}{2\cdot\Delta y}\right) \end{cases}$$

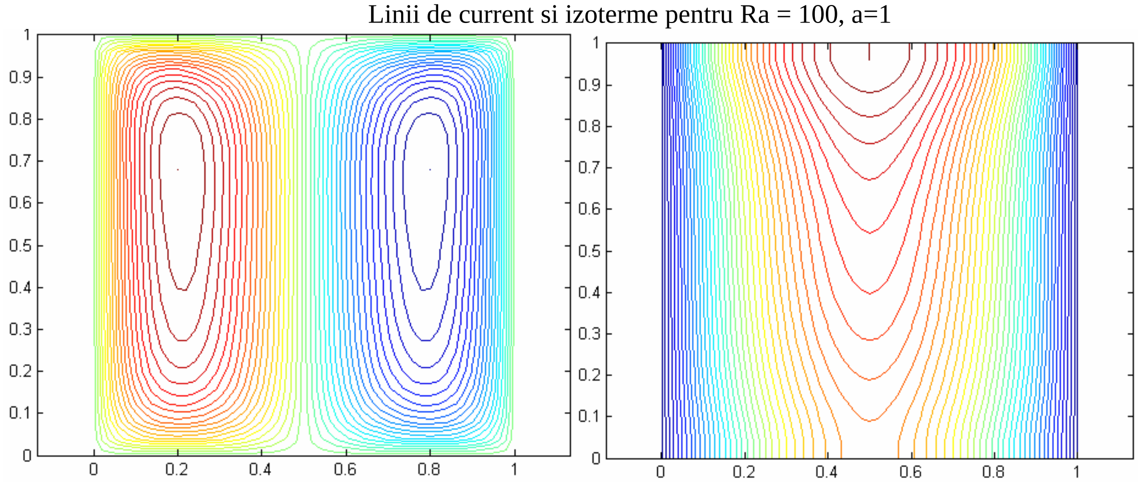
$i = 2, N_x - 1$
$j = 2, N_y - 1$

$$\begin{cases} \Psi_{i,j}\left(\dfrac{2}{(\Delta x)^2} + \dfrac{2a^2}{(\Delta y)^2}\right) = \dfrac{1}{(\Delta x)^2}\left(\tilde{\Psi}_{i+1,j} + \tilde{\Psi}_{i-1,j}\right) + \dfrac{a^2}{(\Delta y)^2}\left(\tilde{\Psi}_{i,j+1} + \tilde{\Psi}_{i,j-1}\right) + \dfrac{Ra}{2\cdot\Delta x}\left(\tilde{\theta}_{i+1,j} - \tilde{\theta}_{i-1,j}\right) \\[2ex] \theta_{ij}\left(\dfrac{2}{(\Delta x)^2} + \dfrac{2a^2}{(\Delta y)^2}\right) = 1 + \dfrac{1}{(\Delta x)^2}\left(\tilde{\theta}_{i+1,j} + \tilde{\theta}_{i-1,j}\right) + \dfrac{a^2}{(\Delta y)^2}\left(\tilde{\theta}_{i,j+1} + \tilde{\theta}_{i,j-1}\right) - \\[2ex] \qquad - \dfrac{a}{4\Delta x \Delta y}\left[\left(\tilde{\Psi}_{i,j+1} - \tilde{\Psi}_{i,j-1}\right)\left(\tilde{\theta}_{i+1,j} - \tilde{\theta}_{i-1,j}\right) - \left(\tilde{\Psi}_{i+1,j} - \tilde{\Psi}_{i-1,j}\right)\left(\tilde{\theta}_{i,j+1} - \tilde{\theta}_{i,j-1}\right)\right] \end{cases}$$

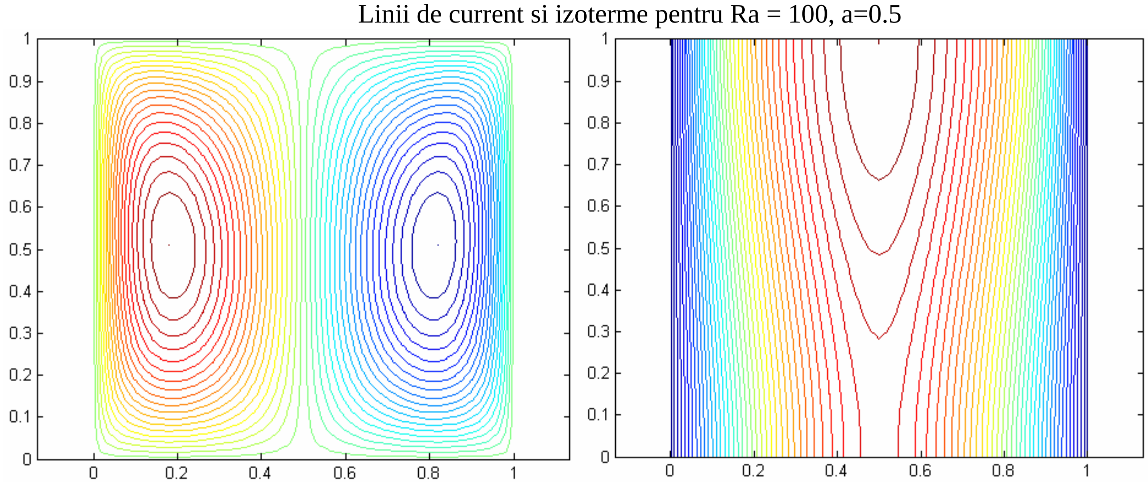
unde „~" semnifică faptul că avem valoarea de la iterația anterioară.



Linii de current si izoterme pentru Ra = 100, a=1



Linii de current si izoterme pentru Ra = 100, a=0.5



Linii de current si izoterme pentru Ra = 100, a=2

Table 1
Accuracy test for $Ra = 10^3$ and $a = 1$

Nodes	$\psi(0.24, 0.24)$	$\theta(0.24, 0.24)$
26×26	2.6368	0.0389
51×51	2.5987	0.0384
101×101	2.5800	0.0382
201×201	2.5707	0.0381
Richardon extrapolation	2.5614	0.0380

Table 2
Comparison of ψ_{max} and θ_{max} for . $a = 0.5$

Ra	Haajizadeh *et al.* [27]		Present (Richardson extrapolation)	
	$\psi_{\max}$	$\theta_{\max}$	$\psi_{\max}$	$\theta_{\max}$
10	0.078	0.130	0.079	0.127
10^3	4.880	0.118	4.833(4.832)	0.116(0.116)

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