# DARBO TYPE BEST PROXIMITY POINT (PAIR) RESULTS USING MEASURE OF NONCOMPACTNESS WITH APPLICATION

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**Abstract.** Primarily this work intends to investigate the existence of best proximity points (pairs) for new classes of cyclic (noncyclic) mappings via simulation functions and measure of noncompactness. Use of different classes of additional functions make it possible to generalize the contractive inequalities in this work. As an application of the main conclusions, a survey for the existence of optimal solutions of a system of integro-differential equations under some new conditions is presented. As an application of our existence results, we establish the existence of a solution for the following system of integro-differential equations

$$\begin{cases} u'(t) = F_1(t, u(t), \int_{t_0}^t k_1(t, s, u(s))ds), & u(t_0) = u_1, \\ v'(t) = F_2(t, v(t), \int_{t_0}^t k_2(t, s, v(s))ds), & v(t_0) = u_2, \end{cases}$$

in the space of all bounded and continuous real functions on $[0, +\infty[$ under suitable assumptions on $F_1, F_2$.

**Key Words and Phrases**: Best proximity point, measure of noncompactness, simulation functions, integro-differential equation, Darbo fixed point theorem.

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