

## ON THE FIXED POINT INDEX FOR SUMS OF OPERATORS

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**Abstract.** We have obtained some new Krasoneslskii type fixed point theorems for the sum of two operators  $T + F$ , where  $T$  is expansive with constant  $h > 1$  and  $I - F$  is a  $k$ -set contraction with  $k < h$ . The existence results are based on a fixed point index for these classes of mappings. Various fixed point theorems are derived in cones and in translates of cones of some Banach spaces. Two examples of application are included to illustrate the theory.

**Key Words and Phrases:** Sum of operators, cone, fixed point index,  $k$ -set contraction, expansive mapping, Krasoneslskii's Theorem.

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### 1. INTRODUCTION

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