

COUPLED HILFER AND HADAMARD FRACTIONAL DIFFERENTIAL SYSTEMS IN GENERALIZED BANACH SPACES

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Abstract. This article deals with some existence and uniqueness of solutions for some coupled systems of Hilfer and Hilfer-Hadamard fractional differential equations. Some applications are made of generalizations of classical fixed point theorems on generalized Banach spaces.

Key Words and Phrases: Fractional differential equation, left-sided mixed Riemann-Liouville integral of fractional order, left-sided mixed Hadamard integral of fractional order, Hilfer fractional derivative, Hadamard fractional derivative, coupled system, generalized Banach space, fixed point.

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