Fixed Point Theory, 23(2022), No. 1, 3-20
DOI: 10.24193/fpt-ro.2022.1.01
http://www.math.ubbcluj.ro/~nodeacj/sfptcj.html

ON SPLIT COMMON FIXED POINT AND MONOTONE INCLUSION PROBLEMS IN REFLEXIVE BANACH SPACES

H.A. ABASS*, A.A. MEBAWONDU**, C. IZUCHUKWU*** AND O.K. NARAIN****

*School of Mathematics, Statistics and Computer Science, University of KwaZulu-Natal, Durban, South Africa
DSI-NRF Center of Excellence in Mathematical and Statistical Sciences (CoE-MaSS)
E-mail: AbassH@ukzn.ac.za, hammedabass548@gmail.com

**School of Mathematics, Statistics and Computer Science, University of KwaZulu-Natal, Durban, South Africa
DSI-NRF Center of Excellence in Mathematical and Statistical Sciences (CoE-MaSS)
E-mail: dele@aims.ac.za

***School of Mathematics, Statistics and Computer Science, University of KwaZulu-Natal, Durban, South Africa
E-mail: izuchukwu_c@yahoo.com

****School of Mathematics, Statistics and Computer Science, University of KwaZulu-Natal, Durban, South Africa
E-mail: naraino@ukzn.ac.za

Abstract. In this paper, we study split common fixed point problems of Bregman demigeneralized and Bregman quasi-nonexpansive mappings in reflexive Banach spaces. Using the Bregman technique together with a Halpern iterative algorithm, we approximate a solution of split common fixed point problem and sum of two monotone operators in reflexive Banach spaces. We establish a strong convergence result for approximating the solution of the aforementioned problems. It is worth mentioning that the iterative algorithm employ in this article is design in such a way that it does not require prior knowledge of operator norm and we do not employ Fejer monotinicity condition in the strategy of proving our convergence theorem. We apply our result to solve variational inequality and convex minimization problems. The result discuss in this paper extends and complements many related results in literature.
Key Words and Phrases: Bregman quasi-nonexpansive, Bregman demigeneralized mapping, monotone operators, fixed point, iterative scheme, split common fixed point problem.
2020 Mathematics Subject Classification: 47H06, 47H09, 47J05, 47J25, 47H10.

Acknowledgement.

The first and second authors acknowledge with thanks the bursary and financial support from Department of Science and Technology and National Research Foundation, Republic of South Africa Center of Excellence in Mathematical and Statistical Sciences (DSI-NRF COE-MaSS) Post-Doctoral Fellowship. Opinions expressed and

conclusions arrived are those of the authors and are not necessarily to be attributed to the CoE-MaSS.

References

[1] H.A. Abass, K.O. Aremu, L.O. Jolaoso, O.T. Mewomo, *An inertial forward-backward splitting method for approximating solutions of certain optimization problem*, J. Nonlinear Funct. Anal., (2020), Article ID 6.

[2] H.A. Abass, C. Izuchukwu, O.T. Mewomo, Q.L. Dong, *Strong convergence of an inertial forward-backward splitting method for accretive operators in real Banach space*, Fixed Point Theory, **20**(2020), no. 2, 397-412.

[3] K. Afassinou, O.K. Narain, O.E. Otunuga, *Iterative algorithm for approximating solutions of split monotone variational inclusion, variational inequality and fixed point problems in real Hilbert spaces*, Nonlinear Funct. Anal. and Appl., **25**(2020), no. 3, 491-510.

[4] A. Akbar, E. Shahrosvand, *Split equality common null point problem for Bregman quasi-nonexpansive mappings*, Filomat, **32**(2018), no. 11, 3917-3932.

[5] Q.H. Ansari, A. Rehan, *Iterative methods for generalized split feasibility problems in Banach spaces*, Carapathian J. Math., **33**(2017), no. 1, 9-26.

[6] K. Aoyama, Y. Kamimura, W. Takahashi, and M. Toyoda, *Approximation of common fixed point of a countable family of nonexpansive mappings in a Banach space*, Nonlinear Anal., **67**(2007), 2350-2360.

[7] K. Aoyama, Y. Kimura, W. Takahashi, M. Toyoda, *Approximation of common fixed points of a countable family of nonexpansive mappings in a Banach space*, Nonlinear Anal., **67**(2007), 2350-2360.

[8] H.H. Bauschke, J.M. Borwein, *Legendre functions and method of random Bregman functions*, J. Convex Anal., **4**(1997), 27-67.

[9] H.H. Bauschke, J.M. Borwein, P.L. Combettes, *Essentially smoothness, essentially strict convexity and Legendre functions in Banach spaces*, Commun. Contemp. Math., **3**(2001), 615-647.

[10] J.Y. Bello Cruz, Y. Shehu, *An iterative method for split inclusion problem without prior knowledge of operator norm*, J. Fixed Point Theory Appl., **19**(2017), no. 3.

[11] E. Blum, W. Oettli, *From optimization and variational inequalities to equilibrium problems*, Math. Stud., **63**(1994), 123-145.

[12] L.M. Bregman, *The relaxation method for finding the common point of convex sets and its application to solution of problems in convex programming*, U.S.S.R. Comput. Math. Phys., **7**(1967), 200-217.

[13] C. Bryne, *Iterative oblique projection onto convex subsets and the split feasibility problems*, Inverse Probl., **18**(2002), 441-453.

[14] M. Burwein, S. Reich, S. Sabach, *A characterization of Bregman firmly nonexpansive operators using a new monotonicity concept*, J. Nonlinear Convex Anal., **12**(2011), 161-184.

[15] D. Butnariu, A.N. Iusem, *Totally Convex Functions for Fixed Points Computation and Infinite Dimensional Optimization*, Kluwer Academic Publishers, Dordrecht, 2000.

[16] D. Butnariu, E. Resmerita, *Bregman distances, totally convex functions and a method for solving operator equations in Banach spaces*, Abstract and Applied Analysis, **2006**, (2006), Art. ID 84919, 1-39.

[17] Y. Censor, T. Elfving, *A multiprojection algorithm using Bregman projections in a product space*, Numer. Algor., **8**(1994), 221-239.

[18] Y. Censor, A. Segal, *The split common fixed point problem for directed operators*, J. Convex Anal., **16**(2009), no. 2, 587-600.

[19] P. Cholamjiak, P. Sunthrayuth, *A Halpern-type iteration for solving the split feasibility problem and fixed point problem of Bregman relatively nonexpansive semigroup in Banach spaces*, Filomat, **32**(2018), no. 9, 3211-3227.

[20] I. Cioranescu, *Geometry of Banach Spaces, Duality Mappings and Nonlinear Problems*, Kluwer Academic Publishers, Dordrecht, 1990.

[21] X.P. Ding, *Perturbed proximal point algorithms for generalized quasi-variational inclusions*, J. Math. Anal. Appl., bf 210(1997), no. 1, 88-101.
[22] H. Gazmeh, E. Naraghirad, *The split common null point problem for Bregman generalized resolvents in two Banach spaces*, Optimization, (2020), DOI:10.1080/02331934.2020.1751157.
[23] H. Iiduka, *Acceleration method for convex optimization over the fixed point set of a nonexpansive mappings*, Math. Prog. Series A, **149**(2015), 131-165.
[24] C. Izuchukwu, C.C. Okeke, F.O. Isiogugu, *A viscosity iterative technique for split variational inclusion and fixed point problems between a Hilbert and a Banach space*, J. Fixed Point Theory Appl., **20**,(2018), no. 157.
[25] G. Kassay, S. Reich, S. Sabach, *Iterative methods for solving systems of variational inequalities in Banach spaces*, SIAM Optim., **21**(2011), 1319-1344.
[26] K.R. Kazmi, R. Ali, S. Yousuf, *Generalized equilibrium and fixed point problems for Bregman relatively nonexpansive mappings in Banach spaces*, J. Fixed Point Theory Appl., **20**(2018), no. 151.
[27] Y. Kimura, S. Saejung, *Strong convergence for a common fixed points of two different generalizations of cutter operators*, Linear Nonlinear Anal., **1**(2015), 53-65.
[28] D. Kinderlehrer, G. Stampacchia, *An Introduction to Variational Inequalities and Their Applications*, Society for Industrial and Applied Mathematics, Philadelphia, 2000.
[29] V. Martin Marquez, S. Reich, S. Sabach, *Bregman strongly nonexpansive operators in reflexive Banach spaces*, J. Math. Anal. Appl., **400**(2013), 597-614.
[30] A. Moudafi, *A note on the split common fixed point problem for quasi-nonexpansive operator*, Nonlinear Anal., **74**(2011), 4083-4087.
[31] F.U. Ogbuisi, C. Izuchukwu, *Approximating a zero of sum of two monotone operators which solves a fixed point problem in reflexive Banach spaces*, Numer. Funct. Anal., **40**(2019), no. 13.
[32] F.U. Ogbuisi, O.T. Mewomo, *Iterative solution of split variational inclusion problem in a real Banach spaces*, Afrika Mat., **28**(2017), 295-309.
[33] F.U. Ogbuisi, O.T. Mewomo, *Iterative solution of split variational inclusion problem in real Banach spaces*, Afrika Mat., **28**(2017), 295-309.
[34] C.C. Okeke, C. Izuchukwu, *Strong convergence theorem for split feasibility problems and variational inclusion problems in real Banach spaces*, Rendiconti de Circolo Matematico di Palermo Series 2, (2020), DOI:10.1007/s12215-020-00508-3.
[35] O.K. Oyewole, H.A. Abass, O.T. Mewomo, *A strong convergence algorithm for a fixed point constraint split null point problem*, Rendiconti de Circolo Matematico di Palermo Series 2, (2020), 1-20.
[36] S. Reich, S. Sabach, *A strong convergence theorem for a proximal-type algorithm in reflexive Banach spaces*, J. Nonlinear Convex Anal., **10**(2009), 471-485.
[37] S. Reich, G. Sabach, *Iterative methods for solving systems of variational inequalities in reflexive Banach spaces*, J. Nonlinear Convex Anal., **10**(2009), 471-485.
[38] S. Reich, S. Sabach, *Two strong convergence theorems for a proximal method in reflexive Banach spaces*, Numer. Funct. Anal. Optim., **31**(2010), 24-44.
[39] R.T. Rockafellar, *Characterization of the subdifferentials of convex functions*, Pacific J. Math., **17**(1966), 497-510.
[40] R.T. Rockafellar, *On the maximality of sums of nonlinear monotone operators*, Trans. Amer. Math. Soc., **149**(1970), 75-88.
[41] F. Schopfer, T. Schuster, A.K. Louis, *An iterative regularization method for the solution of the split feasibilty problem in Banach spaces*, Inverse Probl., **24**(2008), no. 20, 055008.
[42] Y. Shehu, F.U. Ogbuisi, *Approximation of common fixed points of left Bregman strongly nonexpansive mappings and solutions of equilibrium problems*, J. Appl. Anal. **21**(2015), no. 2, 63-77.
[43] Y. Shehu, F.U. Ogbuisi, *An iterative method for solving split monotone variational inclusion and fixed point problem*, RACSAM, **110**(2016), 503-518.
[44] Y. Shehu, F.U. Ogbuisi, O.S. Iyiola, *Convergence analysis of an iterative algorithm for fixed point problems and split feasibility problems in certain Banach spaces*, Optimization, **65** (2016), 299-323.

[45] A. Taiwo, T.O. Alakoya, O.T. Mewomo, *Halpern type iterative process for solving split common fixed point and monotone variational inclusion problem between Banach spaces*, Numer. Algor., (2019), DOI:org/10.1007/s11075-020-00937-2.
[46] J.V. Tie, *Convex Analysis: An Introductory Text*, Wiley, New York, 1984.
[47] S. Timnak, E. Naraghirad, N. Hussain, *Strong convergence of Halpern iteration for products of finitely many resolvents of maximal monotone operators in Banach spaces*, Filomat, **31**(2017), no. 15, 4673-4693.
[48] F.Q. Xia, N.J. Huang, *Variational inclusions with a general H-monotone operators in Banach spaces*, Comput. Math. Appl., **54**(2010), no. 1, 24-30.
[49] C. Zalinescu, *Convex Analysis in General Vector spaces*, World Scientific Publishing Co. Inc., River Edge NJ, 2002.

Received: September 6, 2020; Accepted: July 14, 2021.