FIXED POINT THEOREMS IN r-NORMED AND LOCALLY r-CONVEX SPACES AND APPLICATIONS

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Abstract. In this paper we prove some new fixed point theorems in r-normed and locally r-convex spaces. Our conclusions generalize many well-known results and provide a partial affirmative answer to Schauder's conjecture. Based on the obtained results, we prove the analogue of a Von Neumann's theorem in locally r-convex spaces. In addition, an application to game theory is presented.
Key Words and Phrases: r-normed space, locally r-convex space, s-convex set, fixed point theorems, topological vector space, Schauder's conjecture.
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