

FIXED AND PERIODIC POINT RESULTS IN CONE b -METRIC SPACES OVER BANACH ALGEBRAS; A SURVEY

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Abstract. In this paper, we consider the concept of cone b -metric spaces over Banach algebras and obtain some fixed point results for various definitions of contractive mappings. Moreover, we discuss about the property P and the property Q of fixed point problems. Our results are significant, since we omit the assumptions of normality of cones under which can be generalized and unified a number of recently announced results in the existing literature. In particular, we refer to the results of Huang et al. [H. Huang, G. Deng, S. Radenović, Some topological properties and fixed point results in cone metric spaces over Banach algebras, *Positivity*. (2018), in press].

Key Words and Phrases: Cone b -metric space, Banach algebra, spectral radius, solid cone, fixed point.

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