

## BEST PROXIMITY POINT OF ZAMFIRESCU CONTRACTIONS OF PEROV TYPE ON REGULAR CONE METRIC SPACES

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**Abstract.** T. Zamfirescu, [Fixed point theorems in metric spaces, Arch. Math. (Basel), 23 (1972), 292-298] obtained a very interesting fixed point theorem on complete metric spaces, by combining results of Banach, Kannan and Chatterjea. In this paper, we introduce the concept of Zamfirescu-Perov type cyclic contraction and obtain best proximity point theorems for such mapping in the frame work of regular cone metric spaces. Examples are given to support our results. Our results extend and generalize several comparable existing results in literature.

**Key Words and Phrases:** Cone metric spaces, regular cones, best proximity point, Perov contraction, spectral radius.

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