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MODIFIED INERTIAL METHODS FOR FINDING COMMON SOLUTIONS TO VARIATIONAL INEQUALITY PROBLEMS

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Abstract. It is our aim in this paper to propose modified inertial versions of both subgradient extragradient method and the extragradient method for solving common solutions to variational inequality problems involving monotone and Lipschitz continuous operators and obtain weak convergence results in real Hilbert spaces. We give several numerical illustrations of our proposed methods and give numerical comparisons of our methods with subgradient extragradient and extragradient methods.
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