A GENERAL VISCOSITY IMPLICIT ITERATIVE ALGORITHM FOR SPLIT VARIATIONAL INCLUSIONS WITH HIERARCHICAL VARIATIONAL INEQUALITY CONSTRAINTS

L.C. CENG*, I. COROIAN**, X. QIN*** AND J.C. YAO****,[1]

*Department of Mathematics, Shanghai Normal University
Shanghai 200234, China
E-mail: zenglc@hotmail.com

**University of Agricultural Sciences and Veterinary Medicine
Calea Mănăştur, 3-5, 400372, Cluj-Napoca, Romania
E-mail: coroian.iulia@gmail.com

***Institute of Fundamental and Frontier Sciences
University of Electronic Science and Technology of China
Chengdu, China
E-mail: qxlxajh@163.com

****Department of Mathematics, Zhejiang Normal University
Jinhua, 321004, P. R. China
E-mail: yaojc@mail.cmu.edu.tw

Abstract. The purpose of this paper is to introduce a general viscosity implicit iterative method for finding a solution of a split variational inclusion problem (SVIP) with a hierarchical variational inequality (HVI) constraint for a countable family of nonexpansive mappings in Hilbert spaces. Strong convergence theorem is obtained under some mild assumptions.

Key Words and Phrases: Split variational inclusion, hierarchical variational inequality, nonexpansive mapping, implicit iterative method.

2010 Mathematics Subject Classification: 47H09, 47J20, 47N10.

References

[1] B.A. Bin Dehaish, X. Qin, A. Latif, H.O. Bakodah, *Weak and strong convergence of algorithms for the sum of two accretive operators with applications,* J. Nonlinear Convex Anal., **16**(2015), 1321-1336.

[2] C. Byrne, *Iterative oblique projection onto convex sets and the split feasibility problem,* Inverse Probl., **18**(2002), 441-453.

[3] C. Byrne, *A unified treatment of some iterative algorithms in signal processing and image reconstruction,* Inverse Probl., **20**(2004), 103-120.

[4] C. Byrne, Y. Censor, A. Gibali, S. Reich, *The split common null point problem,* J. Nonlinear Convex Anal., **13**(2012), 759-775.

---

[1]Corresponding author.

[5] L.C. Ceng, Q.H. Ansari, J.C. Yao, *Some iterative methods for finding fixed points and for solving constrained convex minimization problems,* Nonlinear Anal., **74**(2011), 5286-5302.

[6] L.C. Ceng, Q.H. Ansari, J.C. Yao, *Mann type iterative methods for finding a common solution of split feasibility and fixed point problems*, Positivity, **16**(2012), 471-495.

[7] L.C. Ceng, A. Petruşel, J.C. Yao, Y. Yao, *Hybrid viscosity extragradient method for systems for variational inequalities, fixed points of nonexpansive mappings, zero points of accretive operators in Banach spaces,* Fixed Point Theory, **19**(2018), 487-502.

[8] L.C. Ceng, N.C. Wong, J.C. Yao, *Hybrid extragradient methods for finding minimum-norm solutions of split feasibility problems,* J. Nonlinear Convex Anal., **16**(2015), 1965-1983.

[9] Y. Censor, T. Elfving, *A multiprojection algorithm using Bregman projections in a product space,* Numer. Algorithms, **8**(1994), 221-239.

[10] Y. Censor, A. Gibali, S. Reich, *Algorithms for the split variational inequality problem,* Numer. Algorithms, **59**(2012), 301-323.

[11] S.S. Chang, H.W.J. Lee, C.K. Chan, *A new method for solving equilibrium problem fixed point problem and variational inequality problem with application to optimization,* Nonlinear Anal., **70**(2009), 3307-3319.

[12] S.Y. Cho, B.A. Bin Dehaish, X. Qin, *Weak convergence of a splitting algorithm in Hilbert spaces,* J. Appl. Anal. Comput., **7**(2017), 427-438.

[13] S.Y. Cho, X. Qin, L. Wang, *Strong convergence of a splitting algorithm for treating monotone operators,* Fixed Point Theory Appl., **2014**(2014), Art. ID 94.

[14] S.Y. Cho, X. Qin, J.C. Yao, Y. Yao, *Viscosity approximation splitting methods for monotone and nonexpansive operators in Hilbert spaces*, J. Nonlinear Convex Anal., **19**(2018), 251-264.

[15] K.R. Kazmi, S.H. Rizvi, *An iterative method for split variational inclusion problem and fixed point problem for a nonexpansive mapping,* Optim. Lett., **8**(2014), 1113-1124.

[16] G. Marino, H.K. Xu, *A general iterative method for nonexpansive mappings in Hilbert spaces,* J. Math. Anal. Appl., **318**(2006), 43-52.

[17] A. Moudafi, *Split monotone variational inclusions,* J. Optim. Theory Appl., **150**(2011), 275-283.

[18] Z. Opial, *Weak convergence of successive approximations for nonexpansive mappings,* Bull. Amer. Math. Soc., **73**(1967), 591-597.

[19] X. Qin, A. Petruşel, J.C. Yao, *CQ iterative algorithms for fixed points of nonexpansive mappings and split feasibility problems in Hilbert spaces,* J. Nonlinear Convex Anal., **19**(2018), 157-165.

[20] X. Qin, J.C. Yao, *Projection splitting algorithms for nonlinear operators*, J. Nonlinear Convex Anal., **18**(2017), 925-935.

[21] A. Saidi, O. Chadli, J.C. Yao, *Second order nonlinear evolution equations with time dependent pseudomonotone and quasimonotone operators: An equilibrium problem approach,* Appl. Anal. Optim., **1**(2017), 345-359.

[22] K. Shimoji, W. Takahashi, *Strong convergence to common fixed points of infinite nonexpansive mappings and applications,* Taiwanese J. Math., **5**(2001), 387-404.

[23] K. Sitthithakerngkiet, J. Deepho, J. Martinez-Moreno, P. Kumam, *Convergence analysis of a general iterative algorithm for finding a common solution of split variational inclusion and optimization problems,* Numer. Algorithms, doi:10.1007/s11075-017-0462-2.

[24] T. Suzuki, *Strong convergence of Krasnoselskii and Mann's type sequences for one-parameter nonexpansive semigroups without Bochner integrals,* J. Math. Anal. Appl., **305**(2005), 227-239.

[25] W. Takahashi, *Nonlinear Functional Analysis,* Yokohama Publishers, Yokohama, 2000.

[26] W. Takahashi, C.F. Wen, J.C. Yao, *Split common fixed point problems and hierarchical variational inequality problems in Hilbert spaces*, J. Nonlinear Convex Anal., **18**(2017), 777-797.

[27] I. Yamada, *The hybrid steepest-descent method for the variational inequality problems over the intersection of fixed point sets of nonexpansive mappings,* In: D. Butnariu, Y. Censor, S. Reich (eds.), Inherently Parallel Algorithms in Feasibility and Optimization and Their Applications, vol. 8, Studies in Computational Mathematics, pp. 473-505, North-Holland, 2001.

[28] Y. Yao, Y.C. Liou, J.C. Yao, *Finding the minimum norm common element of maximal monotone operators and nonexpansive mappings with involving projection*, J. Nonlinear Convex Anal., **16**(2015), 835-854.