VECTOR EQUILIBRIUM PROBLEMS FOR MULTIFUNCTIONS IN TOPOLOGICAL SEMILATTICE SPACES

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Dedicated to Professor Do Hong Tan on the occasion of his 80th birthday

Abstract. Let K be a nonempty compact Δ-convex subset of a topological semilattice with path-connected intervals. In this paper, under new assumptions, we establish some existence theorems of $x \in K$ such that $\mathcal{F}(A) \cap VEP(f) \neq \emptyset$, where $\mathcal{F}(A)$ is the set of all fixed points of the multifunction $A : K \to 2^K$ and $VEP(f)$ is the set of all solutions for the vector equilibrium problems of the multifunction f from $K \times K$ to a topological vector space Y. These results generalize and improve the recent ones in the literature. Some examples are given to illustrate our results.

Key Words and Phrases: KKM lemma, Ky Fan inequality, Browder-Fan fixed point theorem, multifunction, topological semilattice, C_Δ-quasiconvex (quasiconcave), C-upper (lower) semicontinuous, vector equilibrium problem.

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