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THE IMPLICIT MIDPOINT RULE FOR NONEXPANSIVE MAPPINGS IN BANACH SPACES

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Abstract. The implicit midpoint rule (IMR) for nonexpansive mappings is established in Banach spaces. The IMR generates a sequence by an implicit algorithm. Weak convergence of this algorithm is proved in a uniformly convex Banach space which either satisfies Opial's property or has a Fréchet differentiable norm. Consequently, this algorithm applies in both ℓ_p and L^p for $1 < p < \infty$.
Key Words and Phrases: Implicit midpoint rule, nonexpansive mapping, fixed point, uniformly convex Banach space, Opial's property, Fréchet differentiable norm.
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