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COUPLED FIXED POINT THEOREMS FOR SYMMETRIC CONTRACTIONS IN b-METRIC SPACES WITH APPLICATIONS TO OPERATOR EQUATION SYSTEMS

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Abstract. In this paper, we will consider coupled fixed point problems in b-metric spaces for single-valued operators satisfying a symmetric contraction condition. On one hand, existence and uniqueness of the solution and, on the other hand, data dependence, well-posedness, Ulam-Hyers stability, limit shadowing property of the coupled fixed point problem are discussed. The approach is based on the application of a Ran-Reurings type fixed point theorem for an appropriate operator on the Cartesian product space. Some applications to a system of integral equations and to a periodic boundary value problems are also given.

Key Words and Phrases: Single-valued operator, fixed point, ordered metric space, coupled fixed point, data dependence, well-posedness, Ulam-Hyers stability, limit shadowing, integral equation, periodic boundary value problem.

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