ITERATIVE FIXED POINT THEOREMS AND THEIR APPLICATIONS TO ORDERED VARIATIONAL INEQUALITIES ON VECTOR LATTICES

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Abstract. In this paper, we introduce the concept of order-continuity and ordered Lipschitz conditions of maps on vector lattices, and we provide some properties of order-continuous maps. Then, by applying these properties, we prove some theorems for the existence of fixed points for maps. As applications of these results, we solve some ordered variational inequalities on vector lattices.

Key Words and Phrases: Vector lattice, order-continuity, ordered Lipschitz condition, order-preserving map, fixed point, generalized Archimedean vector lattice, ordered variational inequality.

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References

[1] C.D. Aliprantis, O. Burkinshaw, *Positive Operators*, Springer, 2006.

[2] Y.I. Alber, S. Reich, *An iterative method for solving a class of nonlinear operator equations in Banach spaces*, Panamer. Math. J. , **4**(1994), no. 2, 39-54.

[3] S. Carl, S. Heikkilä, *Fixed Point Theory in Ordered Sets and Applications: From Differential and Integral Equations to Game Theory*, Springer, New York, 2010.

[4] G. Isac, J.L. Li, *Complementarity problems, Karamardian's condition and a generalization of Harker-Pang condition*, Nonlinear Anal. Forum, **6**(2001), no. 5, 383-390.

[5] S. Karamardian, *Generalized complementarity problems*, J. Optim. Theory Appl., **8**(1971), 161-168.

[6] F. Giannessi, *Theorems of Alterative, Quadratic Programs and Complementarity Problems, Variational Inequalities and Complementarity Problems*, (R.W. Cottle, F. Giannessi and J.L. Lions - Eds.), John Wiley and Sons, Chichester, England, 1980, 151-186.

[7] N.J. Huang, Y.P. Fang, *On Vector Variational Inequalities in Reflexive Banach Spaces*, J. Global Optim., **32**(2005), 495-505.

[8] J.L. Li, J.C. Yao, *The existence of maximum and minimum solutions to general variational inequalities in Hilbert lattices*, Fixed Point Theory and Applications, **2011**(2011), 1-19.

[9] J.L. Li, E.A. Ok, *Optimal Solutions to Variational Inequalities on Banach Lattices*, J. Math. Anal. Appl., **388**(2012), 1157-1165.

[10] J.L. Li, *Several Extensions of the Abian-Brown Fixed Point Theorem and Their Applications to Extended and Generalized Nash Equilibria on Chain-Complete Posets*, J. Math. Anal. Appl., **409**(2014), 1084-1002.

[11] J.L. Li, L.S Xie, W.S. Yang, *Vector and ordered variational inequalities and applications to order-optimization problems on Banach lattices*, J. Applied Math., 2013, doi.10.1155 /2013/439394.

[12] E.A. Ok, *Order Theory*, forthcoming.

[13] A.M. Ran, M.C.B. Reurings, *A fixed point theorem in partially ordered sets and some applications to matrix equations*, Proceedings of the AMS, **132**(2003), no. 5, 1435-1443.

[14] M. Renardy, R.C. Rogers, *An Introduction to Partial Differential Equations*, Texts in Applied Mathematics 13, Springer-Verlag, New York 2004.

[15] S. Takahashi, W. Takahashi, *Viscosity approximation methods for equilibrium problems and fixed point problems in Hilbert spaces*, J. Math. Anal. Appl., **331**(2007), 506-515.

[16] A. Tarski, *A lattice-theoretical fixpoint theorem and its applications*, Pacific J. Math., **5**(1955), 285-309.

[17] J.C. Yao, *Variational inequalities with generalized monotone operators*, Math. Oper. Res., **19**(1994), 691-705.