

STRONG CONVERGENCE THEOREMS BY HYBRID METHODS FOR MAXIMAL MONOTONE OPERATORS AND GENERALIZED HYBRID MAPPINGS

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Abstract. Let C be a closed convex subset of a real Hilbert space H . Let T be a supper hybrid mapping of C into H , let A be an inverse strongly monotone mapping of C into H and let B be a maximal monotone operator on H such that the domain of B is included in C . In this paper, we introduce two iterative sequences by hybrid methods of finding a point of $F(T) \cap (A+B)^{-1}0$, where $F(T)$ is the set of fixed points of T and $(A+B)^{-1}0$ is the set of zero points of $A+B$. Then, we prove two strong convergence theorems in a Hilbert space. Using these results, we give some applications.
Key Words and Phrases: Hilbert space, nonexpansive mapping, nonspreading mapping, supper hybrid mapping, fixed point, strong convergence, hybrid method.
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