Fixed Point Theory, 16(2015), No. 1, 185-190
http://www.math.ubbcluj.ro/~nodeacj/sfptcj.html

AN APPLICATION OF THE COMMON FIXED POINT THEOREMS TO THE THEORY OF STABILITY OF FUNCTIONAL EQUATIONS

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Abstract. We present an application of the common fixed point theorems, i.e., Markov-Kakutani fixed point theorem and DeMarr common fixed point theorem, to the stability of the functional equation of the form

$$f(sx) = F(s, f(x)), \quad s \in G,\, x \in X.$$

Key Words and Phrases: Stability of functional equations, common fixed point theorems, homogenity equation, Markov–Kakutani fixed point theorem, DeMarr fixed point theorem.
2010 Mathematics Subject Classification: 39B82, 47H10.

Acknowledgements. This research was supported by the Institute of Mathematics, University of Silesia, Katowice, Poland (*Selected Topics in the Theory of Functional Equations*, programm).

References

[1] J. Aczél, *Lectures on Functional Equations and their Applications*, Academic Press, Vol. 19, New York-London 1966, xx+510 pp.

[2] R.P. Agarwal, B. Xu, W. Zhang, *Stability of functional equations in single variable*, J. Math. Anal. Appl., **288**(2003), no. 2, 852–869.

[3] K. Ciepliński, *Applications of fixed point theorems to the Hyers-Ulam stability of functional equations - a survey*, Ann. Funct. Anal., **3**(2012), no. 1, 151–164.

[4] R. DeMarr, *Common fixed points for isotone mappings*, Colloq. Math., **13**(1964), 45–48.

[5] G.L. Forti, *Comments on the core of the direct method for proving Hyers-Ulam stability of functional equations*, J. Math. Anal. Appl., **295**(2004), 127–133.

[6] G.L. Forti, *Hyers-Ulam stability of functional equations in several variables*, Aequationes Math., **50**(1995), no. 1-2, 143–190.

[7] D.H. Hyers, *On the stability of the linear functional equation*, Proc. Nat. Acad. Sci. U.S.A., **27**(1941), 222–224.

[8] D.H. Hyers, G. Isac, T.M. Rassias, *Stability of Functional Equations in Several Variables; Progress in Nonlinear Differential Equations and their Applications*, Birkhäuser Boston, 34, Inc., Boston, MA, 1998.

[9] D.H. Hyers, T.M. Rassias, *Approximate homomorphisms*, Aequationes Math., **44**(1992), no. 2-3, 125–153.

[10] W. Jabłoński, *Stability of homogeneity almost everywhere*, Acta Math. Hungar., **117**(2007), no. 3, 219–229.

[11] W. Jabłoński, J. Schwaiger, *Stability of the homogeneity and completeness*, Österreich. Akad. Wiss. Math.-Natur. Kl. Sitzungsber. II, **214**(2005), 111–132.

[12] W. Jabłoński, *Stability of the Pexider-type homogeneous equation*, Demonstratio Math., **32**(1999), no. 4, 759–766.

[13] W. Jabłoński, *On the stability of the homogeneous equation*, Publ. Math. Debrecen, **55**(1999), no. 1-2, 33–45.

[14] S.-M. Jung, *Hyers-Ulam-Rassias Stability of Functional Equations in Mathematical Analysis*, Hadronic Press, Inc., Palm Harbor, FL, 2001.

[15] S.-M. Jung, *Hyers-Ulam-Rassias Stability of Functional Equations in Nonlinear Analysis*, Springer, New York, 2011.

[16] S.-M. Jung, *On the superstability of the functional equation* $f(x^y) = yf(x)$, Abh. Math. Sem. Univ. Hamburg, **67**(1997), 315–322.

[17] S. Kakutani, *Two fixed-point theorems concerning bicompact convex sets*, Proc. Imperial Acad. Japan, **14**(1938), 242–245.

[18] Z. Kominek, J. Matkowski, *On stability of the homogeneity condition*, Results in Math., 27 (1995), 373-380.

[19] T.-Ch. Lim, *On the largest common fixed point of a commuting family of isotone maps*, Discrete Contin. Dyn. Syst. 2005, suppl., 621–623.

[20] A. Markov, *Quelques theoremes sur ensembles abeliens*, C.R. (Doklady) Acad. Sci. URSS, N.S., **1**(1936), 311–313.

[21] B. Przebieracz, *The Hyers theorem via the Markov-Kakutani fixed point theorem*, J. Fixed Point Theory Appl., **12**(2012), no. 1-2, 35–39.

[22] Th. M. Rassias, *On the stability of functional equations and a problem of Ulam*, Acta Appl. Math., **62**(2000), no. 1, 23–130.

[23] Th. M. Rassias, *On the stability of functional equations in Banach spaces*, J. Math. Anal. Appl., **251**(2000), no. 1, 264–284.

[24] W. Rudin, *Functional Analysis*. International Series in Pure and Applied Math., McGraw-Hill, Inc., New York, 1991.

[25] L. Székelyhidi, *Note on a stability theorem*, Canad. Math. Bull., **25**(1982), no. 4, 500–501.

[26] L. Székelyhidi, *Remark 17*, Aequationes Math., **29**(1985), 95–96.

[27] L. Székelyhidi, *Ulam's problem, Hyers's solution and to where they led*, Functional Equations and Inequalities, Math. Appl., 518, Kluwer Acad. Publ., Dordrecht, 2000, 259–285.

[28] L. Székelyhidi, *The stability of homogeneous functions*, J. Univ. Kuwait Sci., **20**(1993), no. 2, 159–163.

[29] J. Tabor, J. Tabor, *Homogeneity is superstable*, Publ. Math. Debrecen, **45**(1994), no. 1-2, 123–130.

[30] T. Trif, *On the superstability of certain functional equations*, Demonstratio Math., **35**(2002), no. 4, 813–820.

Received: February 27, 2013; Accepted: March 27, 2013.