W-GPH-KKM THEOREMS IN PSEUDO H-SPACES AND THEIR APPLICATIONS

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Abstract. In this paper, some new W-GPH-KKM theorems are established in pseudo H-spaces without any linear and convex structure under much weaker assumptions, and next as their applications, some new coincidence theorems, maximal element theorems, and existence theorems of solutions to generalized equilibrium problems are proved in pseudo H-spaces. The results represented in this paper unify and extend some corresponding known results in the literature.
Key Words and Phrases: Pseudo H-space, W-GPH-KKM mapping, coincidence theorem, maximal element, generalized equilibrium problem, fixed point.
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