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ON AN A-BIFURCATION THEOREM WITH APPLICATION TO A PARAMETERIZED INTEGRO-DIFFERENTIAL SYSTEM

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Abstract. In this paper, we define a notion of *an A-bifurcation* for a system of differential equations in a separable Hilbert space. By using the methods of the topological degree theory and guiding functions, we prove the theorem on the existence and uniqueness of an A-bifurcation point. It is shown how the abstract result can be applied to study the global structure of the solution set of a feedback control system governed by integro-differential equations.
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References

[1] J.C. Alexander, P.M. Fitzpatrick, *Galerkin approximations in several parameter bifurcation problems*, Math. Proc. Cambridge Philos. Soc., **87**(1980), 489500.

[2] V. Barbu, *Nonlinear Semigroups and Differential Equations in Banach Spaces*. Noordhoff International Publishing, Leyden, 1976.
[3] Y.G. Borisovich, B.D. Gelman, A.D. Myshkis, V.V. Obukhovskii, *Introduction in the Theory of Set-valued Maps and Differential Inclusions*, 2nd edition, Librokom, Moscow, 2011 (in Russian).
[4] K. Borsuk, *Theory of Retracts*, Monografie Mat., vol. 44, PWN, Warsaw, 1967.
[5] F.E. Browder, W.V Petryshyn, *Approximation methods and the generalized topological degree for nonlinear mappings in Banach spaces*, J. Functional Anal., **3**(1969), no. 2, 217-245.
[6] R.E. Gaines, J.L. Mawhin, *Coincidence Degree and Nonlinear Differential Equations* Lecture Notes in Mathematics, no. 568, Springer-Verlag, Berlin-New York, 1977.
[7] L. Górniewicz, A. Granas, W. Kryszewski, *On the homotopy method in the fixed point index theory of multi-valued mappings of compact absolute neighborhood retracts*, J. Math. Anal. Appl., **161**(1991), no. 2, 457-473.
[8] L. Górniewicz, *Topological Fixed Point Theory of Multivalued Mappings*, Topological Fixed Point Theory and Its Applications, vol. 4, Springer, Dordrecht 2006.
[9] Ph. Hartman, *Ordinary Differential Equations.* Corrected reprint of the second (1982) edition [Birkhäuser, Boston, MA]. Classics in Applied Mathematics, 38. SIAM Philadelphia, 2002.
[10] D.M. Hyman, *On decreasing sequences of compact absolute retracts*, Fund. Math., **64**(1969), 91-97.
[11] M. Kamenskii, V. Obukhovskii, P. Zecca, *Condensing Multivalued Maps and Semilinear Differential Inclusions in Banach Spaces*, de Gruyter Series in Nonlinear Analysis and Applications 7, Walter de Gruyter, Berlin-New York, 2001.
[12] M.A. Krasnoselskii, *The Operator of Translation along the Trajectories of Differential Equations*, Translation of Mathematical Monographs, 19, American Mathematical Society, Providence, RI, 1968, 294p.
[13] N.V. Loi, *Method of guiding functions for differential inclusions in a Hilbert space*, Differ. Uravn., **46**(2010), no. 10, 1433-1443 (in Russian); English transl.: Differ. Equat., **46**(2010), no. 10, 1438-1447.
[14] N.V. Loi, V. Obukhovskii, P. Zecca, *Non-smooth guiding functions and periodic solutions of functional differential inclusions with infinite delay in Hilbert spaces*, Fixed Point Theory, **13**(2012), no. 2, 565-582.
[15] I. Massabo, P. Nistri, *A topological degree for multivalued A-proper maps in Banach spaces*, Boll. Un. Mat. Ital., **13-B**(1976), 672-685.
[16] A.D. Myshkis, *Generalizations of the theorem on a fixed point of a dynamical system inside of a closed trajectory*, Mat. Sb., **34**(1954), no. 3, 525-540 (in Russian).
[17] V. Obukhovskii, P. Zecca, N.V. Loi, S. Kornev, *Method of Guiding Functions in Problems of Nonlinear Analysis*, Lecture Notes in Math. 2076, Springer-Velag, 2013.
[18] W.V. Petryshyn, *On the approximation-solvable of equations involving A-proper and pseudo A-proper mappings*, Bull. Amer. Math. Soc., **81**(1975), 223-312.
[19] W.V. Petryshyn, *Generalized Topological Degree and Semilinear Equations*, Cambridge University Press, 1995.
[20] J.R. Webb, S.C. Welsh, *A-proper maps and bifurcation theory*, Lecture Notes in Mathematics 1151, Springer-Verlag 1985, 342-349.
[21] S.C. Welsh, *Bifurcation of A-proper mappings without transversality considerations*, Proc. Roy. Soc. Edinburgh Sect. A, **107**(1-2)(1987), 65-74.
[22] I.A. Rus, A. Petruşel, M.A. Şerban, *Weakly Picard operators: equivalent definitions, applications and open problems*, Fixed Point Theory, **7**(2006), no. 1, 3-22.
[23] I.A. Rus, A. Petruşel, G. Petruşel, *Fixed Point Theory*, Cluj University Press, 2008.
[24] D. O'Regan, N. Shahzad, R.P. Agarwal, *Fixed point theory for generalized contractive maps on spaces with vector-valued metrics*, Fixed Point Theory and Applications, (Eds. Y.J. Cho, J.K. Kim, S. M. Kang), Vol. 6, Nova Sci. Publ., New York, 2007, 143-149.

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