

THE COMMON FIXED POINT SET OF COMMUTING HOLOMORPHIC MAPPINGS IN CARTESIAN PRODUCTS OF BANACH SPACES

MONIKA BUDZYŃSKA*, TADEUSZ KUCZUMOW** AND SIMEON REICH***

*Instytut Matematyki UMCS, 20-031 Lublin, Poland
E-mail: monikab1@hektor.umcs.lublin.pl

**Instytut Matematyki UMCS, 20-031 Lublin, Poland
E-mail: tadek@hektor.umcs.lublin.pl

***Department of Mathematics, The Technion – Israel Institute of Technology
32000 Haifa, Israel
E-mail: sreich@tx.technion.ac.il

Abstract. Using the metric approach to holomorphic fixed point theory, we establish a theorem regarding the structure of the common fixed point set of commuting holomorphic self-mappings of domains in Cartesian products of complex Banach spaces.

Key Words and Phrases: Banach space, Cartesian product, common fixed point set, commuting nonexpansive mappings, fixed point, holomorphic mapping, nonexpansive retract.

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