Fixed Point Theory, 16(2015), No. 1, 15-30
http://www.math.ubbcluj.ro/~nodeacj/sfptcj.html

NULL CONTROLLABILITY FOR A CLASS OF SEMILINEAR DEGENERATE/SINGULAR PARABOLIC EQUATIONS

CUNG THE ANH* AND VU MANH TOI**

*Department of Mathematics
Hanoi National University of Education
136 Xuan Thuy, Cau Giay, Hanoi, Vietnam
E-mail: anhctmath@hnue.edu.vn

**Faculty of Computer Science and Engineering
Hanoi Water Resources University
175 Tay Son, Dong Da, Hanoi, Vietnam
E-mail: toivmmath@gmail.com

Abstract. In this paper we prove the null controllability results for a class of semilinear degenerate/singular one-dimensional parabolic equations. The proof is based on Schauder's fixed point theorem.
Key Words and Phrases: semilinear degenerate/singular parabolic equation, null controllability, Carleman estimates, Hardy-Poincaré type inequality, fixed point.
2010 Mathematics Subject Classification: 35K65, 93B05, 93B07, 47H10.

Acknowledgements. This work is supported by a grant from Vietnam's Ministry of Education and Training.

References

[1] C.T. Anh, T.T.H. Yen, *Finite-dimensional pullback attractors for parabolic equations with Hardy type potentials*, Ann. Pol. Math., **102**(2011), 161-186.
[2] C.T. Anh, V.M. Toi, *Approximate controllability for the semilinear heat equation with an inverse-square potential localized on the boundary*, submitted.
[3] F. Alabau-Boussouira, P. Cannarsa, G. Fragnelli, *Carleman estimates for weakly degenerate parabolic operators with applications to null controllability*, J. Evol. Eq., **6**(2006), 161-204.
[4] P. Baras, J. Goldstein, *The heat equation with a singular potential*, Trans. Amer. Math. Soc., **284**(1984), 121-139.
[5] J. Bebernes, D. Eberly, *Mathematical Problems from Combustion Theory*, Math. Sci. Vol. 83, Springer-Verlag New York, 1989.
[6] J.-M. Buchot, J.-P. Raymond, *A linearized model for boundary layer equations*, in "Optimal Control of Complex Structures" (Oberwolfach, 2000), Internat. Ser. Numer. Math. 139, Birkhauser, Basel, (2002), 31-42.
[7] V.R. Cabanillas, S.B. De Menezes, E. Zuazua, *Null controllability in unbounded domains for the semilinear heat equation with nonlinearities involving gradient terms*, J. Optim. Theory Appl., **110**(2001), 245-264.
[8] X. Cabré, Y. Martel, *Existence versus explosion instantanée pour des équations de lachaleur linéaires avec potentiel singulier*, C.R. Acad. Sci. Paris, **329**(1999), 973-978.

[9] P. Cannarsa, G. Fragnelli, *Null controllability of semilinear degenerate parabolic equations in bounded domains*, Electro. J. Diff. Eq., 2006 (2006), no. 136, 1-20.
[10] P. Cannarsa, G. Fragnelli, J. Vancostenoble, *Linear degenerate parabolic equations in bounded domains: controllability and observability*, Proc. of 22nd IFIP TC 7 Conference on System Modeling and Optimization (Turin, Italy, July 18-22, 2005) (F. Ceragioli, A. Dontchev, H. Furuta, K. Marti,L. Pandolfi,-Eds.).
[11] P. Cannarsa, G. Fragnelli, J. Vancostenoble, *Regional controllability of semilinear degenerate parabolic equations in bounded domains*, J. Math. Anal. Appl., **320**(2006), no. 2, 804-818.
[12] P. Cannarsa, P. Martinez, Vancostenoble, *Persistent regional controllability for a class of degenerate parabolic equations*, Comm. Pure Appl. Anal., **3**(2004), 607-635.
[13] P. Cannarsa, P. Martinez, J. Vancostenoble, *Carleman estimates for a class of degenerate parabolic operators*, SIAM J. Control Optim., **47**(2008), 1-19.
[14] P. Cannarsa, P. Martinez, J. Vancostenoble, *Null controllability of degenerate heat equations*, Adv. Diff. Eq., **10**(2005), 153-190.
[15] S. Ervedoza, *Control and stabilization properties for a singular heat equation with an inverse-square potential*, Comm. in PDE, **33**(2008), 1996-2019.
[16] C. Fabre, J.P. Puel, E. Zuazua, *Approximate controllability of the semilinear heat equation*, Proc. Royal Soc. Edinburgh, **125A**(1995), 31-61.
[17] E. Fernández-Cara, E. Zuazua, *Null and approximate controllability for weakly blowing up semilinear heat equations*, Ann. Inst. H. Poincaré Anal. Non Linéaire, **17**(2000), 583–616.
[18] A. Doubova, E. Fernadez-Cara, M. Gonzalez-Burgos, E. Zuazua, *On the controllability of parabolic systems with a nonlinear term involving the state and the gradient*, SIAM J. Control Optimization, **41**(2002), 798-819.
[19] A.V. Fursikov, O. Yu. Imanuvilov, *Controllability of Evolution Equations*, Lecture Notes Series 34, Research Institute of Mathematics, Global Analysis Research Center, Seoul National University, 1996.
[20] D. Henry, *Geometric Theory of Semilinear Parabolic Equations*, Lecture Notes Math., Vol. 840, Springer, Berlin, 1981.
[21] O. Yu. Imanuvilov, *Controllability of parabolic equations*, Sb. Math., **186**(1995), 109-132 (in Russian).
[22] G. Lebeau, L. Robbiano, *Contrôle exact de l'équation de la chaleur*, Comm. in PDE., **20**(1995), 335-356.
[23] J.-L. Lions, *Quelques méthodes de résolution des problèmes aux limites non linéaires*, Dunod, Paris, 1969.
[24] P. Martinez, J.P. Raymond, J. Vancostenoble, *Regional null controllability for a linearized Crocco type equation*, SIAM J. Control Optim., **42**(2003), no. 2.
[25] L. de Teresa, E. Zuazua, *Approximate controllability of a semilinear heat equation in unbounded domains*, Nonlinear Anal., **37** (1999), 1059–1090.
[26] J. Vancostenoble, *Carleman estimates for one-dimensional degenerate heat equations*, J. Evol. Eq., **6**(2006), 325–362.
[27] J. Vancostenoble, *Improved Hardy-Poincaré inequality and shap Carleman estimates for degenerate/singular parabolic problems*, Disc. Cont. Dyna. Syst. Series S, **4**(2011), 761-790.
[28] J. Vancostenoble, E. Zuazua, *Null controllability for the heat equation with singular inverse-square potentials*, J. Funct. Anal., **254**(2008), 1864-1902.
[29] E. Zuazua, *Finite dimensional null controllability for the semilinear heat equation*, J. Math. Pures et Appl., **76**(1997), 570-594.
[30] E. Zuazua, *Approximate controllability for semilinear heat equations with globally Lipschitz nonlinearities*, Control Cybern., **28**(1999), 665-683.

Received: February 13, 2013; Accepted: July 25, 2013.