

LEVITIN-POLYAK WELL-POSEDNESS OF THE SYSTEM OF WEAK GENERALIZED VECTOR EQUILIBRIUM PROBLEMS

JIAN-WEN PENG* AND JEN-CHIH YAO**

*School of Mathematics, Chongqing Normal University
Chongqing 400047, P. R. China
E-mail: jwpeng2008@gmail.com

**Center for General Education, Kaohsiung Medical University
Kaohsiung 80708, Taiwan, ROC
E-mail: yaojc@math.nsysu.edu.tw

Abstract. In this paper, we introduce two types of the Levitin-Polyak well-posedness for the system of weak generalized vector equilibrium problems. By using the gap function of the system of weak generalized vector equilibrium problems, we establish the equivalent relationship between the two types of Levitin-Polyak well-posedness of the system of weak generalized vector equilibrium problems and the corresponding well-posednesses of the minimization problems. We also present some metric characterizations for the two types of the Levitin-Polyak well-posedness of the system of weak generalized vector equilibrium problems. The results in this paper are new and extend some known results in the literature.

Key Words and Phrases: System of weak generalized vector variational inequalities, Levitin-Polyak well-posedness, Levitin-Polyak approximating solution sequence, gap function, metric characterizations.

2010 Mathematics Subject Classification: 49K40; 47J20, 47H10.

REFERENCES

- [1] Q.H. Ansari, S. Schaible, J.C. Yao, *Systems of Vector Equilibrium problems and its applications*, J. Optim. Theory and Appl., **107**(2000), 547-557.
- [2] Q.H. Ansari, J.C. Yao, *A fixed-point theorem and its applications to the Systems of variational inequalities*, Bull. Austral. Math. Soc., **59**(1999), 433-442.
- [3] J.P. Aubin, I. Ekeland, *Applied Nonlinear Analysis*, John Wiley & Sons, 1984.
- [4] E. Bednarczuk, *Well-posedness of vector optimization problems*, in Lecture Notes in Economics and Mathematical Systems, 294, 51-61, Springer-Verlag, Berlin, 1987.
- [5] M. Bianchi, *Pseudo P -monotone operators and variational inequalities*, Report 6, Istituto di econometria e Matematica per le decisioni economiche, Universita Cattolica del Sacro Cuore, Milan, Italy, 1993.
- [6] L.C. Ceng, J.C. Yao, *Well-posedness of generalized mixed variational inequalities, inclusion problems and fixed-point problems*, Nonlinear Anal., **69**(2008), 4585-4603.

This research was supported by the National Natural Science Foundation of China (Grant No. 11171363 and Grant No. 10831009), the Natural Science Foundation of Chongqing (Grant No. CSTC, 2009BB8240) and the Special Fund of Chongqing Key Laboratory (CSTC 2011KLORSE01).

- [7] G.Y. Chen, X.Q. Yang, H. Yu, *A nonlinear scalarization function and generalized quasi-vector equilibrium problems*, J. Global Optim., **32**(2005), 451-466.
- [8] G. Cohen, F. Chaplais, *Nested monotony for variational inequalities over a product of spaces and convergence of iterative algorithms*, J. Optim. Theory Appl., **59**(1988), 360-390.
- [9] S. Deng, *Coercivity properties and well-posedness in vector optimization*, RAIRO Oper. Res., **37**(2003), 195-208.
- [10] A.L. Dontchev, T. Zolezzi, *Well-posed Optimization Problems*, Springer-Verlag, 1993.
- [11] Y.P. Fang, R. Hu, *Parametric well-posedness for variational inequalities defined by bifunctions*, Comput. Math. Appl., **53**(2007), 1306-1316.
- [12] Y.P. Fang, N.J. Huang, J.C. Yao, *Well-posedness of mixed variational inequalities, inclusion problems and fixed point problems*, J. Global Optim., **41**(2008), 117-133.
- [13] M. Furi, A. Vignoli, *About well-posed optimization problems for functions in metric spaces*, J. Optim. Theory Appl., **5**(1970), 225-229.
- [14] F. Giannessi, *Theorems alternative, Quadratic programs, and complementarity problems*, in: Variational inequalities and complementarity problems, (Eds. R.W. Cottle, F. Giannessi, and J.L. Lions), John Wiley and Sons, New York, 1980, 151-186.
- [15] S.H. Hou, H. Yu, G.Y. Chen, *On system of generalized vector variational inequalities*, J. Global Optim., **40**(2008), 739-749.
- [16] S. Hu, N.S. Papageorgiou, *Handbook of Multivalued Analysis*, Volume I: Theory, Kluwer Academic Publishers, 1997.
- [17] X.X. Huang, *Extended and strongly extended well-posed properties of set-valued optimization problems*, Math. Meth. Oper. Res., **53**(200), 101-116.
- [18] X.X. Huang, *Extended well-posed properties of vector optimization problems*, J. Optim. Theory Appl., **106**(2000), 165-182.
- [19] X.X. Huang, X.Q. Yang, *Generalized Levitin-Polyak well-posedness in constrained optimization*, SIAM J. Optim., **17**(2006), 243-258.
- [20] X.X. Huang, X.Q. Yang, *Levitin-Polyak well-posedness in generalized variational inequalities problems with functional constraints*, J. Ind. Manag. Optim., **3**(2007), 671-684.
- [21] X.X. Huang, X.Q. Yang, *Levitin-Polyak well-posedness of constrained vector optimization problems*, J. Global Optim., **37**(2007), 287-304.
- [22] X.X. Huang, X.Q. Yang, D.L. Zhu, *Levitin-Polyak well-posedness of variational inequalities problems with functional constraints*, J. Global Optim., **44**(2009), 159-174.
- [23] A.S. Konulova, J.P. Revalski, *Constrained convex optimization problems-well-posedness and stability*, Numer. Funct. Anal. Optim., **15**(1994), 889-907.
- [24] C. Kuratowski, *Topologie*, Panstwowe Wydawnictwo Naukowe, Warszawa, Poland, 1952.
- [25] E.S. Levitin, B.T. Polyak, *Convergence of minimizing sequences in conditional extremum problem*, Soviet Mathematics Doklady, **7**(1966), 764-767.
- [26] S.J. Li, M.H. Li, *Levitin-Polyak well-posedness of vector equilibrium problems*, Math. Meth. Oper. Res., **69**(2009), 125-140.
- [27] M.B. Lignola, J. Morgan, *Approximating solutions and α -well-posedness for variational inequalities and Nash equilibria*, in: Decision and Control in Management Science, Kluwer Academic Publ., 2002, 367-378.
- [28] X.J. Long, N.J. Huang, K.L. Teo, *Levitin-Polyak well-posedness for equilibrium problems with functional constraints*, J. Inequal. Appl., Volume 2008, Article ID 657329, 14 pages.
- [29] P. Loridan, *Well-posed Vector Optimization*, Recent Developments in Well-Posed Variational Problems, Mathematics and its Applications, 331, Kluwer Academic Publ., Dordrecht, 1995.
- [30] R. Lucchetti, *Convexity and Well-posed Problems*, Springer, 2006.
- [31] R. Lucchetti, *Well-posedness Towards Vector Optimization*, Lecture Notes in Economics and Mathematical Systems, 294, Springer-Verlag, Berlin, 1987.
- [32] R. Lucchetti, F. Patrone, *A characterization of Tykhonov well-posedness for minimum problems with applications to variational inequalities*, Numer. Funct. Anal. Optim., **3**(4)(1981), 461-476.
- [33] J.S. Pang, *Asymmetric variational inequality problems over product sets: Applications and iterative methods*, Math. Progr., **31**(1985), 206-219.

- [34] J.W. Peng, Y. Wang, S.Y. Wu, *Levitin-Polyak well-posedness of generalized vector equilibrium problems*, Taiwanese J. Math., **15**(2011), 2311-2330.
- [35] J.W. Peng, Y. Wang, L.J. Zhao, *Generalized Levitin-Polyak well-posedness of vector equilibrium problems*, Fixed Point Theory Appl., Volume 2009, Article ID 684304, 14 pages.
- [36] J.W. Peng, S.Y. Wu, *The generalized Tykhonov well-posedness for system of vector quasi-equilibrium problems*, Optim. Lett., **4**(2010), 501-512.
- [37] A.N. Tykhonov, *On the stability of the functional optimization problem*, USSRJ. Comput. Math. Math. Phys. **6**(1966), 28-33.
- [38] Z. Xu, D.L. Zhu, X.X. Huang, *Levitin-Polyak well-posedness in generalized vector variational inequality problem with functional constraints*, Math. Meth. Oper. Res., **67**(2008), 505-524.
- [39] T. Zolezzi, *Extended well-posedness of optimization problems*, J. Optim. Theory Appl., **91**(1996), 257-266.

Received: December 28, 2013; Accepted: March 17, 2013