

THE FIXED POINT PROPERTY IN c_0 WITH THE ALPHA NORM

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Abstract. We study the fixed point and the weak fixed point property in the Banach space $c_{0\alpha} = (c_0, \|\cdot\|_\alpha)$, where $\|(x_i)\|_\alpha = \sup_i |x_i| + \alpha \sum_i \frac{|x_i|}{2^i}$. It is known that $c_{0\alpha}$ has the weak fixed point property for every $\alpha \geq 0$. We prove that if K is a nonempty, convex, closed and bounded subset of $c_{0\alpha}$, then K is weakly compact if and only if every nonempty, convex and closed subset of K has the FPP.

Key Words and Phrases: Fixed point property, space $c_{0\alpha}$.

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