

SOLUTIONS OF A CLASS OF INTERNAL NONLOCAL CAUCHY PROBLEMS FOR THE DIFFERENTIAL EQUATION $x'(t) = f(t, x(t), x'(t))$

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Abstract. In this work, we study the existence of solutions for the Cauchy problem of the differential equation $x'(t) = f(t, x(t), x'(t))$ with some general class of internal nonlocal and integral conditions.
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