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ITERATIVE APPROXIMATION OF SOLUTIONS OF GENERALIZED EQUATIONS OF HAMMERSTEIN TYPE

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Abstract. Let H be a real Hilbert space. For each $i = 1, 2, ...m$, let $F_i,\ K_i : H \to H$ be bounded and monotone mappings. Assume that the generalized Hammerstein equation $u + \sum_{i=1}^{m} K_i F_i u = 0$ has a solution in H. We construct a new explicit iterative sequence and prove strong convergence of the sequence to a solution of the generalized Hammerstein equation. Our iterative scheme in this paper seems far simpler than the iterative scheme used by Chidume and Ofoedu [C. E. Chidume, E. U. Ofoedu; Solution of nonlinear integral equations of Hammerstein type, Nonlinear Anal. 74 (2011), 4293-4299] and Chidume and Shehu [C.E. Chidume, Y. Shehu; Approximation of solutions of generalized equations of Hammerstein type, Comp. Math. Appl. 63 (2012), 966-974].
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