σ -POROUS SETS OF GENERALIZED NONEXPANSIVE MAPPINGS

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Abstract. In the paper we show that many generic results obtained in the last 40 years in the field of fixed point theory, are consequences of one general theorem. As an application we get some extensions of them and we study the generic aspect of the existence of attractors of nonexpansive IFSs. We also give a detailed discussion on the relationship between two results obtained by De Blasi and Myjak, and Reich and Zaslavski.

Key Words and Phrases: Fixed points, porosity, nonexpansive mappings, contractive mappings, set-valued mappings, measures of noncompactness.

2010 Mathematics Subject Classification: 47H10, 54H25, 54E52, 47H08, 47H04.

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