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AN INTEGRAL EQUATION RELATED TO AN EPIDEMIC MODEL VIA WEAKLY PICARD OPERATORS TECHNIQUE IN A GAUGE SPACE

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Abstract. In the paper *Qualitative behavior of an integral equation related to some epidemic model* (Demonstratio Mathematica, Vol. XXXVI, No 3/2003, 603-609) the author Eva Brestovanska has considered the integral equation

$$x(t) = \Big[g_1(t) + \int_0^t A_1(t-s)F_1(s,x(s))ds\Big] \cdots \Big[g_p(t) + \int_0^t A_p(t-s)F_p(s,x(s))ds\Big],\ t \geq 0.$$

In this paper we shall study by weakly Picard technique operators in a gauge space: the existence, uniqueness and data dependence such as the continuity, smooth dependence on parameter for the solution of the following integral equation

$$x(t) = \Big[g_1(t) + \int_0^t K_1(t,s,x(s))ds\Big] \cdot \Big[g_2(t) + \int_0^t K_2(t,s,x(s))ds\Big],\ t \in [0,\infty).$$

Our approach are connected with some results due from I.M. Olaru (*An integral equation via weakly Picard operators*, Fixed Point Theory, Vol 11 No1/2010, 97-106 and *Generalization of an integral equation related to some epidemic models*, Carpathian J. Math. Vol 26, No.1(2010), 92-96).
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