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A MODIFIED HOMOTOPY METHOD FOR SOLVING NONCONVEX FIXED POINTS PROBLEMS

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Abstract. In the paper, to compute the fixed point of a self-mapping in a class of nonconvex unbounded sets with both inequality and equality constraints, a modified homotopy is constructed and the existence and convergence of the smooth homotopy pathways is proved under mild conditions. Compared with the previous results, the modified combined homotopy method needs only weaker conditions. Some numerical examples are given to show the feasibility and effectiveness of the proposed method.

Key Words and Phrases: Combined homotopy method, nonconvex sets, fixed point, pseudo cone condition.

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