

SOLVABILITY OF BOUNDARY VALUE PROBLEMS WITH INTEGRAL CONDITIONS FOR FRACTIONAL DIFFERENTIAL EQUATIONS

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Abstract. In this paper, we study boundary value problems with integral conditions for fractional differential equations of the order $\alpha \in (1, 2)$ in an abstract Banach space. To overcome the difficulty from some mixed integral terms in a fractional integral equation, a necessary Gronwall inequality with some mixed integral terms is established to obtain important a priori bounds. Some sufficient conditions for the existence of solutions are presented by means of fractional calculus and fixed point theorems via different conditions and techniques. An example is given to illustrate the results.

Key Words and Phrases: Fractional differential equations, Boundary value problems, Integral conditions, Generalized Gronwall inequality, Fixed point method.

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REFERENCES

- [1] R.P. Agarwal, M. Benchohra, S. Hamani, *A survey on existence results for boundary value problems of nonlinear fractional differential equations and inclusions*, Acta. Appl. Math., **109**(2010), 973-1033.
- [2] R.P. Agarwal, Y. Zhou, Y. He, *Existence of fractional neutral functional differential equations*, Comput. Math. Appl., **59**(2010), 1095-1100.
- [3] B. Ahmad, J.J. Nieto, *Existence results for nonlinear boundary value problems of fractional integrodifferential equations with integral boundary conditions*, Boundary Value Prob., **2009**(2009), Article ID 708576, e1-e11.
- [4] B. Ahmad, J.J. Nieto, *Existence results for a coupled system of nonlinear fractional differential equations with three-point boundary conditions*, Comput. Math. Appl., **58**(2009), 1838-1843.
- [5] B. Ahmad, J.J. Nieto, *Existence of solutions for anti-periodic boundary value problems involving fractional differential equations via Leray-Schauder degree theory*, Topol. Methods Nonlinear Anal., **35**(2010), 295-304.
- [6] J.P. Aubin, I. Ekeland, *Applied Nonlinear Analysis*, Wiley-Interscience, 1984.
- [7] Z. Bai, *On positive solutions of a nonlocal fractional boundary value problem*, Nonlinear Anal., **72**(2010), 916-924.
- [8] M. Benchohra, J. Henderson, S.K. Ntouyas, A. Ouahab, *Existence results for fractional order functional differential equations with infinite delay*, J. Math. Anal. Appl., **338**(2008), 1340-1350.
- [9] Y.-K. Chang, J.J. Nieto, *Some new existence results for fractional differential inclusions with boundary conditions*, Math. Comput. Model., **49**(2009), 605-609.
- [10] C. Crăciun, N. Lungu, *Abstract and concrete Gronwall lemmas*, Fixed Point Theory, **10**(2009), 221-228.
- [11] K. Diethelm, *The Analysis of Fractional Differential Equations*, Lecture Notes in Mathematics, 2010.
- [12] A. Granas, J. Dugundji, *Fixed Point Theory*, Springer, New York, 2003.
- [13] J. Henderson, A. Ouahab, *Fractional functional differential inclusions with finite delay*, Nonlinear Anal., **70**(2009), 2091-2105.
- [14] A.A. Kilbas, H.M. Srivastava, J.J. Trujillo, *Theory and Applications of Fractional Differential Equations*, in: North-Holland Mathematics Studies, vol. 204, Elsevier Science B.V., Amsterdam, 2006.
- [15] N. Lungu, *Qualitative Problems in the Theory of Hyperbolic Differential Equations*, Digital Data, Cluj-Napoca, 2005.
- [16] N. Lungu, *On some Volterra integral inequalities*, Fixed Point Theory, **8**(2007), 39-45.
- [17] K.S. Miller, B. Ross, *An Introduction to the Fractional Calculus and Differential Equations*, John Wiley, New York, 1993.
- [18] V. Lakshmikantham, S. Leela, J. V. Devi, *Theory of Fractional Dynamic Systems*, Cambridge Scientific Publishers, 2009.
- [19] I. Podlubny, *Fractional Differential Equations*, Academic Press, San Diego, 1999.
- [20] I.A. Rus, *Gronwall lemma approach to the Hyers-Ulam-Rassias stability of an integral equation*, in: Nonlinear Analysis and Variational Problems, Springer, 2009, 147-152.
- [21] I.A. Rus, *Gronwall lemmas: ten open problems*, Sci. Math. Jpn., **70**(2009), 221-228.
- [22] D.R. Smart, *Fixed Point Theorems*, Cambridge University Press, Cambridge, 1980.
- [23] V.E. Tarasov, *Fractional Dynamics: Application of Fractional Calculus to Dynamics of Particles, Fields and Media*, Springer, HEP, 2010.
- [24] J. Wang, X. Xiang, Y. Peng, *Periodic solutions of semilinear impulsive periodic system on Banach space*, Nonlinear Anal., **71**(2009), 1344-1353.
- [25] J. Wang, W. Wei, *A class of nonlocal impulsive problems for integrodifferential equations in Banach spaces*, Results Math., **58**(2010), 379-397.
- [26] J. Wang, Y. Zhou, W. Wei, *Impulsive fractional evolution equations and optimal controls in infinite dimensional spaces*, Topol. Meth. Nonlinear Anal., **38**(2011), 17-43.
- [27] J. Wang, Y. Zhou, *A class of fractional evolution equations and optimal controls*, Nonlinear Anal., **12**(2011), 262-272.

- [28] J. Wang, L. Lv, Y. Zhou, *Ulam stability and data dependence for fractional differential equations with Caputo derivative*, Electron. J. Qualitative Theory Diff. Eq., 2011, no. 63, 1-10.
- [29] J. Wang, Y. Zhou, W. Wei, H. Xu, *Nonlocal problems for fractional integrodifferential equations via fractional operators and optimal controls*, Comp. Math. Appl., **62**(2011), 1427-1441.
- [30] J. Wang, Y. Zhou, W. Wei, *A class of fractional delay nonlinear integrodifferential controlled systems in Banach spaces*, Commun. Nonlinear Sci. Numer. Simulat., **16**(2011), 4049-4059.
- [31] J. Wang, Y. Zhou, *Existence and controllability results for fractional semilinear differential inclusions*, Nonlinear Anal., **12**(2011), 3642-3653.
- [32] J. Wang, Y. Zhou, *Analysis of nonlinear fractional control systems in Banach spaces*, Nonlinear Anal., **74**(2011), 5929-5942.
- [33] J. Wang, Y. Zhou, *Existence of mild solutions for fractional delay evolution systems*, Appl. Math. Comput., **218**(2011), 357-367.
- [34] J. Wang, L. Lv, Y. Zhou, *New concepts and results in stability of fractional differential equations*, Commun. Nonlinear Sci. Numer. Simulat., (2011), doi:10.1016/j.cnsns.2011.09.030.
- [35] J. Wang, Y. Zhou, *Mittag-Leffler-Ulam stabilities of fractional evolution equations*, Appl. Math. Lett., (2011), doi:10.1016/j.aml.2011.10.009.
- [36] Y. Zhou, F. Jiao, J. Li, *Existence and uniqueness for p -type fractional neutral differential equations*, Nonlinear Anal., **71**(2009), 2724-2733.
- [37] Y. Zhou, F. Jiao, *Nonlocal Cauchy problem for fractional evolution equations*, Nonlinear Anal., **11**(2010), 4465-4475.

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