

MULTIPLICITY OF CONCAVE AND MONOTONE POSITIVE SOLUTIONS FOR NONLINEAR FOURTH-ORDER ORDINARY DIFFERENTIAL EQUATIONS

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Abstract. By constructing an equivalent operator and using fixed point theorem, multiple positive solutions for some fourth-order multi-point boundary value problems with nonlinearity depending on all order derivatives are obtained. The associated Green's functions are also given.

Key Words and Phrases: Multi-point boundary value problem, positive solution, cone, fixed point.

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