EXISTENCE AND UNIQUENESS OF SOLUTIONS TO A SYSTEM OF FUNCTIONAL EQUATIONS AND APPLICATIONS TO PARTIAL METRIC SPACES

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Abstract. In this paper, we discuss the existence and uniqueness of solutions to the system of functional equations:

$$\begin{cases} Tx = x \\ \varphi(x) = 0 \end{cases}$$

where $T : X \to X$ is a given mapping and $\varphi : X \to [0, \infty)$ is a lower semi-continuous function on X endowed with a metric d. We apply our obtained results to derive some fixed point theorems on partial metric spaces. This answers three open problems posed by Ioan A. Rus in [Fixed point theory in partial metric spaces, Anal. Univ. de Vest, Timisoara, Seria Matematică-Informatică. 46 (2) (2008) 141-160].

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