VISCOSITY APPROXIMATION METHOD FOR EQUILIBRIUM AND FIXED POINT PROBLEMS

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Abstract. In this paper, we introduce a new iterative scheme by the viscosity approximation method for finding a common element of the set of solutions of an equilibrium problem and the set of common fixed points of infinitely many nonexpansive mappings in a Hilbert space. Then, we prove a strong convergence theorem which improves and extends some recent results.
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