

## POSITIVE SOLUTION FOR SECOND ORDER MULTI-POINT BOUNDARY VALUE PROBLEM AT RESONANCE

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**Abstract.** By using Leggett-Williams norm-type theorem due to O'Regan and Zima, we establish the existence of positive solution for a class of second-order  $m$ -point boundary value problem at resonance.

**Key Words and Phrases:**  $M$ -point boundary value problem, resonance, positive solution, cone, fixed point.

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