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ON THE COMPLETENESS OF ORDERED SETS

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Abstract. In this work, we introduce a density property in ordered sets that is weaker than the order density. Then, we prove a strong version of a result proved by Büber and Kirk, which is a special case of the Brouwer Reduction Theorem, in metric spaces relating completeness and density of ordered sets.
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