APPROXIMATE JORDAN DERIVATIONS
ON HILBERT C^*- MODULES

FRIDOUN MORADLOU* AND MADJID ESHAGHI GORDJI**

* Department of Mathematics, Sahand University of Technology, Tabriz, Iran
E-mail: moradlou@sut.ac.ir

**Department of Mathematics, Semnan University, P. O. Box 35195-363, Semnan, Iran
E-mail: meshaghi@semnan.ac.ir

Abstract. In this paper we prove the generalized Hyers–Ulam–Rassias stability of Jordan derivations on Hilbert C^*- modules associated with the following generalized Jensen type functional equation

$$2 \sum_{j=1}^{n} f\left(\frac{x_j}{2} + \sum_{i=1, i \neq j}^{n} x_i \right) + \sum_{i=1}^{n} f(x_j) = 2n\, f\left(\sum_{i=1}^{n} x_i \right).$$

Key Words and Phrases: Hilbert C^*- modules, Hyers–Ulam–Rassias stability, Jordan derivation.
2010 Mathematics Subject Classification: 39B72, 39B82, 47H10, 46L05, 46B03.

References

[1] J. Aczél and J. Dhombres, *Functional Equations in Several Variables,* Cambridge University Press, 1989.

[2] T. Aoki, *On the stability of the linear transformation in Banach spaces*, J. Math. Soc. Japan, **2**(1950), 64–66.

[3] C. Borelli and G.L. Forti, *On a general Hyers–Ulam stability result,* Internat. J. Math. Math. Sci., **18**(1995), 229–236.

[4] D.G. Bourgin, *Classes of transformations and bordering transformations,* Bull. Amer. Math. Soc., **57**(1951), 223–237.

[5] L. Cădariu and V. Radu, *Fixed points and the stability of Jensen's functional equation*, J. Inequal. Pure Appl. Math., **4**(2003), no. 1, Art. 4.

[6] L. Cădariu and V. Radu, *Fixed points and the stability of quadratic functional equations*, An. Univ. Timişoara, Ser. Mat. Inform., **41**(2003), 25–48.

[7] L. Cădariu and V. Radu, *On the stability of the Cauchy functional equation: a fixed point approach*, Grazer Math. Ber., **346**(2004), 43–52.

[8] S. Czerwik, *The stability of the quadratic functional equation*, in: Stability of mappings of Hyers–Ulam type (Th.M. Rassias, J. Tabor - Eds.), Hadronic Press, Florida, 1994, 81–91.

[9] J. Diaz and B. Margolis, *A fixed point theorem of the alternative for contractions on a generalized complete metric space*, Bull. Amer. Math. Soc., **74**(1968), 305–309.

[10] M. Eshaghi Gordji, *Nearly involutions on Banach algebras; A fixed point approach*, Fixed Point Theory, accepted.

[11] M. Eshaghi Gordji, H. Khodaei, *Solution and stability of generalized mixed type cubic, quadratic and additive functional equation in quasi–Banach spaces*, Nonlinear Anal., **71**(2009), 5629–5643.

[12] M. Eshaghi Gordji, J. M. Rassias and N. Ghobadipour, *Generalized Hyers-Ulam Stability of generalized (N,K)-derivations*, Abstract Applied Anal., vol. 2009 (2009), Art. ID 437931.

[13] M. Eshaghi Gordji, M.B. Ghaemi, S. Kaboli Gharetapeh, S. Shams and A. Ebadian, *On the stability of J*−derivations*, J. Geometry and Physics, **60**(2010), no. 3, 454-459.

[14] M. Eshaghi Gordji and A. Najati, *Approximately J*−homomorphisms: A fixed point approach*, J. Geometry and Physics, **60**(2010), no. 5, 809-814.

[15] R. Farokhzad and S.A.R. Hosseinioun, *Perturbations of Jordan higher derivations in Banach ternary algebras: An alternative fixed point approach,* Internat. J. Nonlinear Anal. Appl., **1**(2010), no. 1, 42–53.

[16] P. Găvruta, *A generalization of the Hyers–Ulam–Rassias stability of approximately additive mappings*, J. Math. Anal. Appl., **184**(1994), 431–436.

[17] P. Găvruta and L. Găvruta, *A new method for the generalized Hyers-Ulam-Rassias stability*, Int. J. Nonlinear Anal. Appl. **1**(2010), no. 2, 11–18.

[18] M. B. Ghaemi and B. Alizadeh, *Superstability of higher derivations on Hilbert C*−modules,* Internat. J. Nonlinear Anal. Appl., **1**(2010), no. 2, 36–43.

[19] D.H. Hyers, *On the stability of the linear functional equation*, Proc. Nat. Acad. Sci. U.S.A., **27**(1941), 222–224.

[20] K. Jun and Y. Lee, *On the Hyers–Ulam–Rassias stability of a Pexiderized quadratic inequality,* Math. Inequal. Appl., **4**(2001), 93–118.

[21] S.-M. Jung and J.M. Rassias, *A fixed point approach to the stability of a functional equation of the spiral of Theodorus,* Fixed Point Theory and Applications, to appear.

[22] I. Kaplansky, *Modules over Operator Algebras,* Amer. Math. Soc., Providence, **RI** 1997.

[23] H. Khodaei and Th.M. Rassias, *Approximately generalized additive functions in several variables*, Int. J. Nonlinear Anal. Appl., **1**(2010), no. 1, 22–41.

[24] G.H. Kim, *On the stability of quadratic mapping in normed spaces*, Internat. J. Math. Math. Sci., **25**(2001), 217–229.

[25] H.-M. Kim and J.M. Rassias, *Generalization of Ulam stability problem for Euler-Lagrange quadratic mappings,* J. Math. Anal. Appl., **336**(2007), 277-296.

[26] Y.-S. Lee and S.-Y. Chung, *Stability of an Euler-Lagrange-Rassias equation in the spaces of generalized functions,* Applied Math. Letters, doi:10.1016/j.aml.2007.07.022, 1-7.

[27] L. Maligranda, *A result of Tosio Aoki about a generalization of Hyers−Ulam stability of additive functions - a question of priority,* Aequationes Math., **75**(2008), 289–296.

[28] F. Moradlou, H. Vaezi and C. Park, *Fixed points and stability of an additive functional equation of n-Apollonius type in C*-algebras,* Abstract Applied Anal., vol. 2008, Article ID 672618, 13 pages, 2008. doi:10.1155/2008/672618.

[29] F. Moradlou, A. Najati and H. Vaezi, *Stability of homomorphisms and derivations on C*-ternary rings associated to an Euler–Lagrange type additive mapping,* Result. Math., **55**(2009), 469-486.

[30] A. Najati, *Hyers–Ulam stability of an n−Apollonius type quadratic mapping,* Bull. Belg. Math. Soc. Simon Stevin, **14**(2007), 755-774.

[31] P. Nakmahachalasint, *On the generalized Ulam–Găvruta–Rassias stability of mixed–type linear and Euler–Lagrange–Rassias functional equations,* Internat. J. Math. Math. Sci., **2007** (2007), Art. ID 63239.

[32] C. Park, *On the stability of the linear mapping in Banach modules*, J. Math. Anal. Appl., **275**(2002), 711–720.

[33] C. Park, *On an approximate automorphism on a C*−algebra*, Proc. Amer. Math. Soc., **132**(2004), 1739–1745.

[34] C. Park, *Homomorphisms between Poisson JC*−algebras*, Bull. Braz. Math. Soc., **36**(2005), 79–97.

[35] C. Park, *Fixed points and Hyers–Ulam–Rassias stability of Cauchy-Jensen functional equations in Banach algebras*, Fixed Point Theory and Applications, **2007** (2007), Art. ID 50175 .

[36] C. Park, *Stability of an Euler–Lagrange–Rassias type additive mapping*, Internat. J. Appl. Math. Stat., **7**(2007), 101–111.

[37] C. Park, *Generalized Hyers–Ulam–Rassias stability of quadratic functional equations: a fixed point approach*, Fixed Point Theory Appl., **2008** (2008), Art. ID 493751.

[38] C. Park, *Hyers–Ulam–Rassias stability of a generalized Apollonius–Jensen type additive mapping and isomorphisms between $C^*-algebras$*, Math. Nachr., **281**(2008), 402–411.

[39] C. Park and J. M. Rassias, *Stability of the Jensen-type functional equation in $C^*-algebras$: A fixed point approach*, Abstract Applied Anal., **2009** (2009), Art. ID 360432.

[40] C. Park and Th.M. Rassias, *Hyers–Ulam stability of a generalized Apollonius type quadratic mapping,* J. Math. Anal. Appl., **322**(2006), 371–381.

[41] C. Park and Th.M. Rassias, *Homomorphisms in $C^*--ternary$ algebras and $JB^*-triples,$* J. Math. Anal. Appl., **337**(2008), 13–20.

[42] A. Pietrzyk, *Stability of the Euler–Lagrange–Rassias functional equation,* Demonstratio Math., **39**(2006), 523–530.

[43] V. Radu, *The fixed point alternative and stability of functional equations*, Fixed Point Theory, **1**(2003), 91–96.

[44] J.M. Rassias, *On approximation of approximately linear mappings by linear mappings*, J. Funct. Anal., **46**(1982), 126–130.

[45] J.M. Rassias, *On approximation of approximately linear mappings by linear mappings*, Bull. Sci. Math., **108**(1984), 445–446.

[46] J.M. Rassias, *Solution of a problem of Ulam*, J. Approx. Theory, **57**(1989), 268–273.

[47] J.M. Rassias, *Alternative contraction principle and Ulam stability problem*, Math. Sci. Res. J., **9**(2005), 190–199.

[48] J.M. Rassias, *Alternative contraction principle and alternative Jensen and Jensen type mappings,* Internat. J. Appl. Math. Stat., **4**(2006), 1-10.

[49] J.M. Rassias and M.J. Rassias, *Refined Ulam stability for Euler–Lagrange type mappings in Hilbert spaces,* Internat. J. Appl. Math. Stat., **7**(2007), 126–132.

[50] Th.M. Rassias, *On the stability of the linear mapping in Banach spaces*, Proc. Amer. Math. Soc., **72**(1978), 297–300.

[51] Th.M. Rassias, *The problem of S.M. Ulam for approximately multiplicative mappings*, J. Math. Anal. Appl., **246**(2000), 352–378.

[52] Th.M. Rassias, *On the stability of functional equations in Banach spaces*, J. Math. Anal. Appl., **251**(2000), 264–284.

[53] F. Skof, *Local properties and approximations of operators,* Rend. Sem. Mat. Fis. Milano, **53**(1983), 113-129.

[54] S.M. Ulam, *A Collection of the Mathematical Problems*, Interscience Publ. New York, 1960.