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FIXED POINTS, LIE $*$–HOMOMORPHISMS AND LIE $*$–DERIVATIONS ON LIE C^*–ALGEBRAS

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Abstract. In this paper, using fixed point methods we investigate Lie $*$–homomorphisms between Lie C^*–algebras, and Lie $*$–derivations on Lie C^*–algebras associated with the generalized Jensen–type functional equation

$$\mu f\left(\frac{\sum_{i=1}^{n} x_i}{n}\right)+\mu\sum_{j=2}^{n} f\left(\frac{\sum_{i=1,i\neq j}^{n} x_i-(n-1)x_j}{n}\right)-f(\mu x_1)=0.$$

Key Words and Phrases: Approximate Lie $*$–homomorphism, approximate Lie $*$–derivation, Lie C *–algebra, alternative fixed point.
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