RELAXED IMPLICIT EXTRAGRADIENT-LIKE METHODS
FOR FINDING MINIMUM-NORM SOLUTIONS
OF THE SPLIT FEASIBILITY PROBLEM

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Abstract. In this paper, we consider the split feasibility problem (SFP) in infinite-dimensional Hilbert spaces, and study the relaxed implicit extragradient-like methods for finding a common element of the solution set Γ of the SFP and the set Fix(S) of fixed points of a nonexpansive mapping S. Combining Mann's implicit iterative method and Korpelevich's extragradient method, we propose two implicit iterative algorithms for finding an element of Fix(S) ∩ Γ. On one hand, for $S = I$, the identity mapping, we derive the strong convergence of one implicit iterative algorithm to the minimum-norm solution of the SFP under appropriate conditions. On the other hand, we also derive the weak convergence of another implicit iterative algorithm to an element of Fix(S) ∩ Γ under mild assumptions.
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