FIXED POINTS AND FRACTIONAL DIFFERENTIAL EQUATIONS: EXAMPLES

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Abstract. We study a fractional differential equation of Caputo type by first inverting it as an integral equation, then noting that the kernel is completely monotone, and finally transforming it into another integral equation with a kernel which supports both contractions and compact maps. That kernel allows us to use fixed point theory to obtain qualitative properties of solutions. At the end of Section 4 we give a list of five transformations which convert challenging problems into simple fixed point problems. We treat linear, superlinear, and sublinear examples using Krasnoselskii's fixed point theorem.
Key Words and Phrases: fractional differential equations, integral equations, fixed points.
2010 Mathematics Subject Classification: 34A08, 47G05, 34D20, 47H10.

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