

FIXED POINT THEOREMS FOR NONSELF SINGLE-VALUED ALMOST CONTRACTIONS

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Abstract. Let X be a Banach space, K a non-empty closed subset of X and let $T : K \rightarrow X$ be a non-self almost contraction. The main result of this paper shows that if T has the so called property (M) and satisfies Rothe's boundary condition, i.e., maps ∂K (the boundary of K) into K , then T has a fixed point in K . This theorem generalizes several fixed point theorems for non-self mappings and also extends several important results in the fixed point theory of self mappings to the case on non-self mappings.

Key Words and Phrases: Banach space; non self almost contraction; fixed point; property (M)

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