

## NEARLY INVOLUTIONS ON BANACH ALGEBRAS. A FIXED POINT APPROACH

MADJID ESHAGHI GORDJI

Department of Mathematics, Semnan University, P. O. Box 35195-363, Semnan, Iran  
E-mail: madjid.eshaghi@gmail.com

**Abstract.** Using fixed point methods, we investigate the Hyers–Ulam–Rassias stability and super-stability of involutions on Banach algebras. Moreover, we show that under some conditions on an approximate involution, the Banach algebra has a  $C^*$ -algebra structure.

**Key Words and Phrases:** Hyers–Ulam–Rassias stability; superstability; involution;  $C^*$ -algebra

**2010 Mathematics Subject Classification:** 46L06,46L05,46L35,39B82

**Acknowledgements.** The author would like to extend his thanks to referee for his (her) valuable comments and suggestions which helped simplify and improve the results of paper.

### REFERENCES

- [1] J.A. Baker, *The stability of certain functional equations*, Proc. Amer. Math. Soc., **112**(1991), 729–732.
- [2] L. Cădariu, V. Radu, *On the stability of the Cauchy functional equation: a fixed point approach*, Grazer Mathematische Berichte, **346**(2004), 43–52.
- [3] L. Cădariu, V. Radu, *The fixed points method for the stability of some functional equations*, Carpathian J. Math., **23**(2007), 63–72.
- [4] L. Cădariu, V. Radu, *Fixed points and the stability of quadratic functional equations*, Analele Universităţii de Vest din Timişoara, **41**(2003), 25–48.
- [5] L. Cădariu, V. Radu, *Fixed points and the stability of Jensen’s functional equation*, J. Inequal. Pure Appl. Math., **4**(2003), Article ID 4.
- [6] P.W. Cholewa, *Remarks on the stability of functional equations*, Aequat. Math., **27**(1984), 76–86.
- [7] S. Czerwik, *On the stability of the quadratic mapping in normed spaces*, Abh. Math. Sem. Univ. Hamburg, **62**(1992), 59–64.
- [8] S. Czerwik, *Functional Equations and Inequalities in Several Variables*, World Scientific, London, 2002.
- [9] A. Ebadian, A. Najati, M. Eshaghi Gordji, *On approximate additive–quartic and quadratic–cubic functional equations in two variables on abelian groups*, Results. Math., **58**(2010), no. 1–2, 39–53.
- [10] M. Eshaghi Gordji, M. B. Ghaemi, S. Kaboli Gharetapeh, S. Shams, A. Ebadian, *On the stability of  $J^*$ -derivations*, Journal of Geometry and Physics, **60**(2010), no. 3, 454–459.
- [11] M. Eshaghi Gordji, H. Khodaei, *Stability of Functional Equations*, LAP LAMBERT Academic Publishing, 2010.
- [12] M. Eshaghi Gordji, A. Najati, *Approximately  $J^*$ -homomorphisms: A fixed point approach*, Journal of Geometry and Physics, **60**(2010), 809–814.

- [13] Z. Gajda, *On stability of additive mappings*, Internat. J. Math. Math. Sci., **14**(1991), 431–434.
- [14] P. Găvruta, *A generalization of the Hyers-Ulam-Rassias stability of approximately additive mappings*, J. Math. Anal. Appl., **184**(1994), 431–436.
- [15] L.Găvruta, *Matkowski contractions and Hyers-Ulam stability*, Bul. St. Univ. "Politehnica" Timisoara, Mat. Fiz., **53**(2008), no. 2, 32–35.
- [16] P. Găvruta, L. Găvruta, *A new method for the generalized Hyers-Ulam-Rassias stability*, Int. J. Nonlinear Anal. Appl., **1**(2010), 11–18.
- [17] M.E. Gordji, H. Khodaei, *Solution and stability of generalized mixed type cubic, quadratic and additive functional equation in quasi-Banach spaces*, Nonlinear Anal., **71**(2009), 5629–5643.
- [18] D.H. Hyers, *On the stability of the linear functional equation*, Proc. Natl. Acad. Sci. USA, **27**(1941), 222–224.
- [19] D.H. Hyers, G. Isac, Th.M. Rassias, *Stability of Functional Equations in Several Variables*, Birkhäuser, Basel, 1998.
- [20] S.M. Jung, *Hyers-Ulam-Rassias Stability of Functional Equations in Mathematical Analysis*, Hadronic Press Inc., Palm Harbor, Florida, 2001.
- [21] S.M. Jung, *Hyers-Ulam-Rassias stability of Jensen's equation and its application*, Proc. Amer. Math. Soc., **126**(1998), 3137–3143.
- [22] S.M. Jung, *Stability of the quadratic equation of Pexider type*, Abh. Math. Sem. Univ. Hamburg, **70**(2000), 175–190.
- [23] H. Khodaei, Th.M. Rassias, *Approximately generalized additive functions in several variables*, Int. J. Nonlinear Anal. Appl., **1**(2010), 22–41.
- [24] B. Margolis, J.B. Diaz, *A fixed point theorem of the alternative for contractions on the generalized complete metric space*, Bull. Amer. Math. Soc., **126**(1968), 305–309.
- [25] D. Mihet, *The Hyers-Ulam stability for two functional equations in a single variable*, Banach J. Math. Anal. Appl., **2**(2008), no. 1, 48–52.
- [26] G.J. Murphy,  *$C^*$ -Algebras and Operator Theory*, Academic Press. Inc., 1990.
- [27] C. Park, *Isomorphisms between unital  $C^*$ -algebras*, J. Math. Anal. Appl., **307**(2005), 753–762.
- [28] C. Park, A. Najati, *Homomorphisms and derivations in  $C^*$ -algebras*, Abst. Appl. Anal., **2007**(2007), Article ID 80630.
- [29] V. Radu, *The fixed point alternative and the stability of functional equations*, Fixed Point Theory, **4**(2003), no. 1, 91–96.
- [30] Th.M. Rassias, *On the stability of the linear mapping in Banach spaces*, Proc. Amer. Math. Soc., **72**(1978), 297–300.
- [31] Th.M. Rassias, *New characterization of inner product spaces*, Bull. Sci. Math., **108**(1984), 95–99.
- [32] Th.M. Rassias, P. Šemrl, *On the behaviour of mappings which do not satisfy Hyers-Ulam stability*, Proc. Amer. Math. Soc., **114**(1992), 989–993.
- [33] I.A. Rus, A. Petrusel, G. Petrusel, *Fixed Point Theory*, Cluj University Press, 2008, 514 pp.
- [34] S.M. Ulam, *Problems in Modern Mathematics*, Chapter VI, Science Ed. Wiley, New York, 1940.

*Received: February 2, 2011; Accepted: October 10, 2011.*

