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COMMON COUPLED FIXED POINT RESULTS FOR HYBRID NONLINEAR CONTRACTIONS IN METRIC SPACES

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Abstract. The concept of triangular function is introduced and two kind of hybrid nonlinear contractions involving a gauge function and a triangular function are considered. Several new common coupled fixed point theorems are established in complete metric spaces, and error estimates for iterations to approximate a fixed point are given. The presented results are general because the triangular function is abstract. As applications the existence and uniqueness of the common coupled solutions for a differential system and a integral system are proved respectively.
Key Words and Phrases: Metric space, hybrid nonlinear contraction, gauge function, coupled fixed point, error estimate.
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References

[1] R.P. Agarwal, M.A. El-Gebeily, D. O'Regan, *Generalized contractions in partially ordered metric spaces*, Appl. Anal., **87**(2008), 1-8.
[2] V. Berinde, *Generalized coupled fixed point theorems for mixed monotone mappings in partially ordered metric spaces*, Nonlinear Anal., **74**(2011), 7347-7355.
[3] T.G. Bhaskar, V. Lakshmikantham, *Fixed point theorems in partially ordered metric spaces and applications*, Nonlinear Anal., **65**(2006), 1379-1393.
[4] D.W. Boyd, J.S. Wong, *On nonlinear contractions*, Proc. Amer. Math. Soc., **20**(1969), 458-464.
[5] K. Goebel, W.A. Kirk, *Topics in Metric Fixed Point Theory*, Cambridge University Press, Cambridge, 1990.
[6] A. Granas, J. Dugundji, *Fixed Point Theory*, Springer-Verlag, New York, 2003.
[7] J. Harjani, B. López, K. Sadarangani, *Fixed point theorems for mixed monotone operators and applications to integral equations*, Nonlinear Anal., **74**(2011), 1749-1760.
[8] J. Harjani, K. Sadarangani, *Generalized contractions in partially ordered metric spaces and applications to ordinary differential equations*, Nonlinear Anal., **72**(2010), 1188-1197.
[9] J. Jachymski, *Equivalent conditions for generalized contractions on (ordered) metric spaces*, Nonlinear Anal., **74**(2011), 768-774.

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[10] J. Jachymski, *Equivalence of some contractivity properties over metrical structures*, Proc. Amer. Math. Soc., **125**(1997), 2327-2335.
[11] S.N. Ješić, D. O'Regan, N.A. Babačev, *A common fixed point theorem for R-weakly commuting mappings in probabilistic spaces with nonlinear contractive conditions*, Appl. Math. Comput., **201**(2008), 272-281.
[12] E. Karapınar, *Coupled fixed point theorems for nonlinear contractions in cone metric spaces*, Comput. Math. Appl., **59**(2010), 3656-3668.
[13] V. Lakshmikantham, L. Ćirić, *Coupled fixed point theorems for nonlinear contractions in partially ordered metric spaces*, Nonlinear Anal., **70**(2009), 4341-4349.
[14] N.V. Luong, N.X. Thuan, *Coupled fixed points in partially ordered metric spaces and application*, Nonlinear Anal., **74**(2011), 983-992.
[15] D. O'Regan, A. Petruşel, *Fixed point theorems for generalized contractions in ordered metric spaces*, J. Math. Anal. Appl., **341**(2008), 1241-1252.
[16] M.-D. Rus, *Fixed point theorems for generalized contractions in partially ordered metric spaces with semi-monotone metric*, Nonlinear Anal., **74**(2011), 1804-1813.
[17] R. Saadati, S.M. Vaezpour, *Monotone generalized weak contractions in partially ordered metric spaces*, Fixed Point Theory, **11**(2010), 375-382.
[18] B. Samet, *Coupled fixed point theorems for a generalized Meir-Keeler contraction in partially ordered metric spaces*, Nonlinear Anal., **72**(2010), 4508-4517.
[19] J.-Z. Xiao, G. Li, *Foundations of Abstract Analysis*, Tsinghua University Press, Beijing, 2009.

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