

CAUCHY PROBLEMS FOR FRACTIONAL DIFFERENTIAL EQUATIONS VIA PICARD AND WEAKLY PICARD OPERATORS TECHNIQUE

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Abstract. In this paper we study existence, uniqueness and data dependence for the solutions of Cauchy problems for fractional differential equations in Banach spaces by using Picard and weakly Picard operators technique and suitable Bielecki norms in some convenient spaces.

Key Words and Phrases: Fractional differential equations, Picard operators, weakly Picard operators.

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REFERENCES

- [1] R.P. Agarwal, M. Benchohra, S. Hamani, *A survey on existence results for boundary value problems of nonlinear fractional differential equations and inclusions*, Acta. Appl. Math., **109**(2010), 973-1033.
- [2] R.P. Agarwal, M. Belmekki, M. Benchohra, *A survey on semilinear differential equations and inclusions involving Riemann-Liouville fractional derivative*, Adv. Difference Equ., 2009, Article ID 981728, 1-47.
- [3] R.P. Agarwal, Y. Zhou, Y. He, *Existence of fractional neutral functional differential equations*, Comput. Math. Appl., **59**(2010), 1095-1100.
- [4] B. Ahmad, J.J. Nieto, *Existence results for a coupled system of nonlinear fractional differential equations with three-point boundary conditions*, Comput. Math. Appl., **58**(2009), 1838-1843.

- [5] B. Ahmad, J.J. Nieto, *Existence of solutions for anti-periodic boundary value problems involving fractional differential equations via Leray-Schauder degree theory*, Topol. Methods Nonlinear Anal., **35**(2010), 295-304.
- [6] K. Balachandran, J.Y. Park, *Nonlocal Cauchy problem for abstract fractional semilinear evolution equations*, Nonlinear Anal., **71**(2009), 4471-4475.
- [7] K. Balachandran, S. Kiruthika, J.J. Trujillo, *Existence results for fractional impulsive integrodifferential equations in Banach spaces*, Commun. Nonlinear Sci. Numer. Simulat., **16**(2011), 1970-1977.
- [8] Z. Bai, *On positive solutions of a nonlocal fractional boundary value problem*, Nonlinear Anal., **72**(2010), 916-924.
- [9] M. Benchohra, J. Henderson, S.K. Ntouyas, A. Ouahab, *Existence results for fractional order functional differential equations with infinite delay*, J. Math. Anal. Appl., **338**(2008), 1340-1350.
- [10] M.M. El-Borai, *Some probability densities and fundamental solutions of fractional evolution equations*, Chaos Solitons Fractals, **14**(2002), 433-440.
- [11] Y.-K. Chang, J. J. Nieto, *Some new existence results for fractional differential inclusions with boundary conditions*, Math. Comput. Model., **49**(2009), 605-609.
- [12] K. Diethelm, *The Analysis of Fractional Differential Equations*, Lecture Notes in Mathematics, 2010.
- [13] J. Henderson, A. Ouahab, *Fractional functional differential inclusions with finite delay*, Nonlinear Anal., **70**(2009), 2091-2105.
- [14] E. Hernández, D. O'Regan, K. Balachandran, *On recent developments in the theory of abstract differential equations with fractional derivatives*, Nonlinear Anal., **73**(2010), 3462-3471.
- [15] A.A. Kilbas, H.M. Srivastava, J.J. Trujillo, *Theory and Applications of Fractional Differential Equations*, in: North-Holland Mathematics Studies, vol. 204, Elsevier Science B.V., Amsterdam, 2006.
- [16] V. Lakshmikantham, S. Leela, J.V. Devi, *Theory of Fractional Dynamic Systems*, Cambridge Scientific Publishers, 2009.
- [17] K.S. Miller, B. Ross, *An introduction to the fractional calculus and differential equations*, John Wiley, New York, 1993.
- [18] V. Mureşan, *Existence, uniqueness and data dependence for the solutions of some integrodifferential equations of mixed type in Banach space*, J. Anal. Appl., **23**(2004), 205-216.
- [19] G.M. N'Guérékata, *A Cauchy problem for some fractional differential abstract differential equation with nonlocal conditions*, Nonlinear Anal., **70**(2009), 1873-1876.
- [20] G.M. Mophou, G.M. N'Guérékata, *Existence of mild solutions of some semilinear neutral fractional functional evolution equations with infinite delay*, Appl. Math. Comput., **216**(2010), 61-69.
- [21] I. Podlubny, *Fractional differential equations*, Academic Press, San Diego, 1999.
- [22] I.A. Rus, *Metrical fixed point theorems*, Univ. of Cluj-Napoca, Romania, 1979.
- [23] I.A. Rus, *Picard mappings: results and problems*, Preprint, Cluj-Napoca, Babeş-Bolyai Univ., Seminar on Fixed Point Theory, Preprint, **6**(1987), 55-64.
- [24] I.A. Rus, *Weakly Picard mappings*, Comment. Math. Univ. Carolinae, **34**(1993), 769-773.
- [25] I.A. Rus, S. Mureşan, *Data dependence of the fixed points set of some weakly Picard operators*, In: Proc. Itinerant Seminar (Elena Popoviciu editor, Srima Publishing House), Cluj-Napoca (Romania): Babeş-Bolyai Univ., 2000, 201-207.
- [26] I.A. Rus, *Picard operators and applications*, Scientiae Mathematicae Japonicae, **58**(2003), 191-219.
- [27] M.A. Şerban, I.A. Rus, A. Petruşel, *A class of abstract Volterra equations, via weakly Picard operators technique*, Math. Ineq. Appl., **13**(2010), 255-269.
- [28] V.E. Tarasov, *Fractional dynamics: Application of fractional calculus to dynamics of particles, fields and media*, Springer, HEP, 2010.
- [29] J. Wang, Y. Zhou, W. Wei, *Impulsive fractional evolution equations and optimal controls in infinite dimensional spaces*, Topol. Methods Nonlinear Anal., **38**(2011), 17-43.
- [30] J. Wang, Y. Zhou, *A class of fractional evolution equations and optimal controls*, Nonlinear Anal., **12**(2011), 262-272.

- [31] J. Wang, L. Lv, Y. Zhou, *Ulam stability and data dependence for fractional differential equations with Caputo derivative*, Electron. J. Qualitative Theory of Diff. Eq., 2011, No. 63, e1-e10.
- [32] J. Wang, Y. Zhou, W. Wei, H. Xu, *Nonlocal problems for fractional integrodifferential equations via fractional operators and optimal controls*, Comp. Math. Appl., **62**(2011), 1427-1441.
- [33] J. Wang, Y. Zhou, W. Wei, *A class of fractional delay nonlinear integrodifferential controlled systems in Banach spaces*, Commun. Nonlinear Sci. Numer. Simulat., **16**(2011), 4049-4059.
- [34] J. Wang, Y. Zhou, *Existence and controllability results for fractional semilinear differential inclusions*, Nonlinear Anal., **12**(2011), 3642-3653.
- [35] J. Wang, Y. Zhou, *Analysis of nonlinear fractional control systems in Banach spaces*, Nonlinear Anal., **74**(2011), 5929-5942.
- [36] J. Wang, Y. Zhou, *Existence of mild solutions for fractional delay evolution systems*, Appl. Math. Comput., **218**(2011), 357-367.
- [37] Y. Zhou, F. Jiao, J. Li, *Existence and uniqueness for p-type fractional neutral differential equations*, Nonlinear Anal., **71**(2009), 2724-2733.
- [38] Y. Zhou, F. Jiao, J. Li, *Existence and uniqueness for fractional neutral differential equations with infinite delay*, Nonlinear Anal., **71**(2009), 3249-3256.
- [39] Y. Zhou, F. Jiao, *Existence of mild solutions for fractional neutral evolution equations*, Comp. Math. Appl., **59**(2010), 1063-1077.
- [40] Y. Zhou, F. Jiao, *Nonlocal Cauchy problem for fractional evolution equations*, Nonlinear Anal., **11**(2010), 4465-4475.

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