

## A SPLITTING-RELAXED PROJECTION METHOD FOR SOLVING THE SPLIT FEASIBILITY PROBLEM

FENGHUI WANG

Department of Mathematics, Luoyang Normal University  
Luoyang 471022, P.R. China  
E-mail: wfenghui@gmail.com

**Abstract.** The split feasibility problem (SFP) is to find  $x \in C$  so that  $Ax \in Q$ , where  $C$  is a nonempty closed convex subset of  $\mathbb{R}^n$ ,  $Q$  is a nonempty closed convex subset of  $\mathbb{R}^m$ , and  $A$  is a matrix from  $\mathbb{R}^n$  into  $\mathbb{R}^m$ . One of successful methods for solving the SFP is Byrne's CQ algorithm. However, to carry out the CQ algorithm, it is required that the closed convex subsets are simple and that the matrix norm is known in advance. Motivated by Tseng's splitting method and Yang's relaxed CQ algorithm, we propose in this paper a new method for solving the SFP, which overcomes the drawback of the CQ algorithm.

**Key Words and Phrases:** Split feasibility problem, relaxed projection, CQ algorithm, splitting method.

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