

AN EFFICIENT ALGORITHM FOR SOLVING HIGH ORDER STURM-LIOUVILLE PROBLEMS USING VARIATIONAL ITERATION METHOD

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Abstract. In this paper, a novel numerical algorithm based on generalized variational iteration method for the solution of every $2m$ -order Sturm-Liouville problem for $m \geq 1$ is proposed. In this approach, a Lagrange multiplier is identified to establish suitable correction functional to construct an approximate solution which it is considered as the fixed point of the corresponding correction functional. It is proved that this algorithm converges to the corresponding exact solution. Error estimate for the algorithm is given. Numerical simulations show that this algorithm is easy to implement and produces accurate results. Numerical results are given.

Key Words and Phrases: Sturm-Liouville problems, Lagrange multiplier, eigenvalues, eigenfunctions, variational iteration method.

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