

ON FIXED POINTS OF POINTWISE LIPSCHITZIAN TYPE MAPPINGS

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Abstract. The classes of pointwise Lipschitzian type mappings and asymptotic pointwise Lipschitzian type mappings are introduced and their properties are discussed. The existence of fixed points for pointwise Lipschitzian type mappings and asymptotic pointwise Lipschitzian type mappings are investigated in Banach spaces. Our results generalize and improve the corresponding results of L.P. Belluce and W.A. Kirk [Fixed point theorems for certain classes of nonexpansive mappings, *Proc. Amer. Math. Soc.*, 20 (1969) 141-146]; E. Casini and E. Maluta [Fixed points of uniformly Lipschitzian mappings in spaces with uniformly normal structure, *Nonlinear Anal.*, 9 (1985), 103-108]; K. Goebel and W.A. Kirk [A fixed point theorem for asymptotically nonexpansive mappings, *Proc. Amer. Math. Soc.*, 35 (1972), 171-174]; W.A. Kirk and H.K. Xu [Asymptotic pointwise contractions, *Nonlinear Anal.*, 69 (2008), 4706-4712]; and T.C. Lim and H.K. Xu [Fixed point theorems for asymptotically nonexpansive mappings, *Nonlinear Anal.*, 22(11) (1994), 1345-1355].

Key Words and Phrases: fixed point, asymptotic center, asymptotically nonexpansive mapping, nearly Lipschitzian mapping, point-wise contraction.

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