Fixed Point Theory, 14(2013), No. 1, 161-170
http://www.math.ubbcluj.ro/~nodeacj/sfptcj.html

A FIXED-POINT APPROACH OF A PARACHUTE PROBLEM

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Abstract. The object of this work is to determine the existence of that shape of a parachute, which would cause it to float indefinitely in an ascendant wind stream, even while subject to gravity. A Helmholtz type model is constructed for the unsteady, inviscid, incompressible and potential flow past the parachute. The associated complex potential is determined by making certain reasonable simplifying assumptions and the global action of the fluid (air) on the parachute is evaluated. The existence of a shape of the parachute that would result in its failure, i.e., floating indefinitely, is then determined using a fixed-point technique. A similar conclusion could be get for certain bucket shape of a wind turbine, which leads to its immobility irrespective of the wind stream. This is a problem of practical interest for parachute (turbine) manufacturers, as such a shape should be avoided.
Key Words and Phrases: Helmholtz model for heavy inviscid incompressible flows, parachute in an ascendant wind stream, indefinitely floating parachute, fixed point technique.
2000 Mathematics Subject Classification: 76B10, 35R35, 47H10.

1. Introduction

Suppose that the fluid stream is unlimited at far field (A), the ascendant fluid stream past the curvilinear obstacle (parachute) CC' has a (given) velocity $\mathbf{V}_\infty(t)$, the whole configuration belonging to the Helmholtz model with two separation (jet) lines, λ and λ', emanating from the edges (C and C') of the curvilinear obstacle. Obviously the determination of such a flow can not be separated from the problem of determining the jet lines whose shape is “a priori” unknown, i.e. we are dealing with an inverse problem.

The obstacle is supposed to be symmetrical versus the Ox axis, along this axis, but in opposite sense, the gravity force is acting too. The origin of this axis $O \equiv B$ corresponds to the stagnation point for the ”attack” stream of velocity $\mathbf{V}_\infty(t)$. This

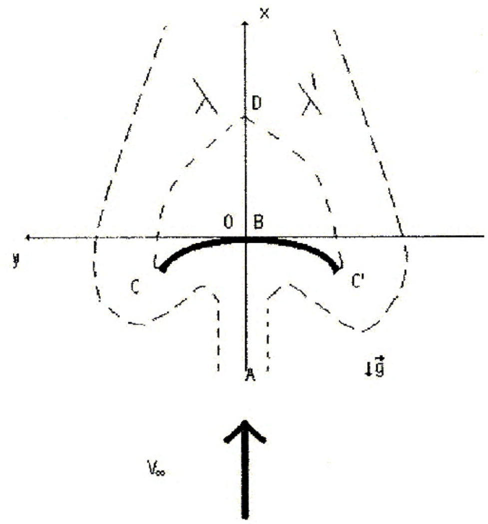

FIGURE 1

symmetrical feature will allow us to study the flow only in the half-plane $y \geq 0$ (see Figure 1).

Problems of this type were in attention of many scientists [1], [2], [3], [4], [5] but all these authors did take into consideration only the steady case in the absence of gravity.

In what follows we will get an analytical approximate solution of this problem based upon an approximate solution of the Bernoulli integral considered on a jet (separation) line [9].

2. Mathematical model for the envisaged fluid flow

Concerning the separation (jet) lines, λ and λ',whose analyticity has been shown by Lewy, while Brillouin has proved their convexity towards the non-cavity fluid domain [6], they will be the borders of a cavity zone of constant pressure p_0.

Denoting by $f(z;t) = \varphi(x,y;t) + i\psi(x,y;t)$ and by $w(z;t) = u - iv = Ve^{-i\omega}$ the complex potential and the complex velocity respectively in the physical plane $z = x + iy$ (where φ is the potential velocity, ψ is the stream function, u and v are the components of the flow velocity of magnitude V and incidence ω), by p and ρ the pressure and the mass density respectively, the Bernoulli integral on the trajectory $ABC\lambda$ is

$$\frac{\partial \varphi}{\partial t} + \frac{p}{\rho} + \frac{1}{2}V^2 + gx = c(t),$$

$c(t)$ being an arbitrary function of time. Accepting that

$$f(0;t) = \varphi(0,0;t) + i\psi(0,0;t) = 0$$

($z = 0$ corresponding to the stagnation point $O \equiv B$), then we have $c(t) = \frac{p_s^0}{\rho}$, where p_s^0 is the pressure at the stagnation point $z = 0$.

We remark that under the hypothesis that the parachute is quasi-flat in the neighborhood of its edges $C(C')$ - which is related to the assumption that the slope and the modulus of the fluid velocity in $\mathcal{V}(C)$ are free time constants, the quantity p_s^0 could be considered time constant too [8].

From the previous Bernoulli integral considered only on the jet (separation) line λ when the pressure $p = p_0$, we have

$$V^2 = 2\left[\frac{p_s^0 - p_0}{\rho} - \frac{\partial \varphi}{\partial t} - gx\right],$$

where $V^2 = (\varphi_x)^2 + (\varphi_y)^2$, i.e., the potential velocity satisfies a first order non linear partial differential equation. Let us look for a solution of this equation under the form

$$\varphi^0 = tH(x, y) + F(x, y),$$

where we impose the fulfilment of the following partial differential equations

$$H_x'^2 + H_y'^2 = 0,\ H_x' F_x' + H_y' F_y' = 0.$$

Hence $H \equiv a$, what leads to the fulfillment of the following partial differential equation

$$F_x'^2 + F_y'^2 = -2\left[a - \frac{p_s^0 - p_0}{\rho} + gx\right] \equiv l(x),$$

where a is a negative constant ($a \leq \frac{p_s^0 - p_0}{\rho} < 0$). We remark that the compulsory positiveness of $l(x)$ (V^2) is synonymous with $-\frac{p_s^0 - p_0}{\rho} + gx \leq -a$ what fails irrespective of how we choose the negative constant a, for an x positive and sufficiently great. Consequently there is another point D, placed by symmetry reasons downstream on the Ox axis, where the velocity vanishes. This second "stagnation point", of abscissa $x_D = \frac{p_s^0 - p_0}{\rho} - \frac{a}{g}$ will "close" the backwards obstacle cavity zone, our configuration belonging to the case when the separation lines are crossing (such configurations with cross jet lines have been already considered [6], [7]).

Let us now introduce the following representation for $F(x, y)$ on λ[9]

$$F(x, y) = \cos f(y) \int_{x_D}^{x} \sqrt{l(x)}dx + \sqrt{l(x)} \int_0^y \sin f(y) dy,$$

i.e., $F_x' = \sqrt{l(x)} \cos f(y)$, and $F_y' = \sqrt{l(x)} \sin f(y)$, where for $f(y)$ we could accept $f(y) = A(y - y_C) + \theta_C$ where A is a positive constant, θ_C is the backwards flow velocity incidence within the "flat zone". At the same time the regularity conditions on fluid flow (incompressibility and irrotationality) lead to $l(x) = e^{-2Ax}$ for $x \neq 0, x_D$, while the representation $f(y) = A(y - y_C) + \theta_C$ shows up that f is a monotonous increasing function for $y \in (y_C, 0)$ what fits to physical considerations.

Now we could directly determine the shape of the "a priori" unknown jet line. To make precise, the jet line, time invariable, is among the solutions of the differential equation

$$\frac{dx}{\sqrt{l(x)}\cos\left[A(y-y_C)+\theta_C\right]}=\frac{dy}{\sqrt{l(x)}\sin\left[A(y-y_C)+\theta_C\right]}$$

and consequently $\sin\left[A(y-y_C)+\theta_C\right]=\sin\theta_C e^{A(x-x_C)}$ where for $y=0$ we should have $x=x_D$ and immediately we get an expression for the positive constant A [9].

Once found the jet (separation) lines for determining the fluid flow in the physical plane we will consider, firstly, the image of our physical flow domain of the plane (z) into the superior half-plane $\psi \geq 0$ of the complex potential $f=\varphi+i\psi$ plane and then a conformal mapping of Joukovski type which maps the domain of the plane f onto a unit superior half-disk, centered at the origin, of a new plane $\tau=\xi+i\eta$, whose semi-circumference corresponds to the jet lines. The images of some main points of the physical plane in these two auxiliary planes (f) and (τ) are configured out as below, see Figure 2 (b being a positive constant which will be determined from some regularity requirements for our solution [8]).

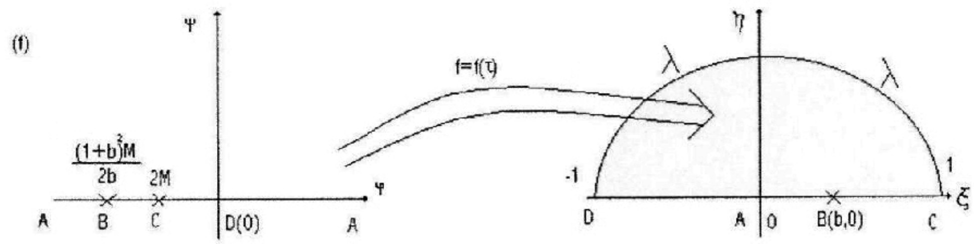

Figure 2

Finally we are led to the following B.V.P. in the plane (τ):

"To determine, in the unit semi-disk of the plane (τ), a function $g^*(\tau)$ such that its derivative satisfies the equality

$$\frac{dg^*}{d\tau}=\left[(\tau+1)^{\alpha}(b-\tau)^{1/2}+(\tau+1)^{\alpha+\varepsilon}\right]\frac{dg}{d\tau},$$

where α and ε are positive, less than unity exponents, observing the restriction $0<\alpha+\varepsilon<1$, while $\frac{dg}{d\tau}$ is an analytical function, on the same closed semi-disk, fulfilling the following requirements

$$\operatorname{Re}\left\{\log\frac{dg}{d\tau}\right\}|_{CD}=\ln\frac{\sqrt{l(x)}}{L_\tau},$$

$$\operatorname{Im}\left\{\log\frac{dg}{d\tau}\right\}|_{AB\cup DA}=0$$

$$\operatorname{Im}\left\{\log\frac{dg}{d\tau}\right\}|_{BC}=y'(x),$$

$y=y(x)$, $x\in(0,x_C)$, being the equation of our "half" obstacle, and

$$L_\tau=\left|(\tau+1)^{\alpha}(b-\tau)^{1/2}+(\tau+1)^{\alpha+\varepsilon}\right|_{|\tau|=1,\eta>0} \quad [9].$$

The above B.V.P. is in fact a mixed problem of Volterra type which can be reduced to a Dirichlet problem for half-plane in a classical way [8]. Using this technique we get [9].

$$g^*(\tau) = \int_{-1}^{\tau} \left[(\tau+1)^{\alpha}(b-\tau)^{1/2} + (\tau+1)^{\alpha+\varepsilon}\right] \cdot$$
$$\cdot \exp\left(\frac{1}{\pi}\sqrt{P(f)}\int_{\frac{(1+b)^2 M}{2b}}^{0} \frac{\mu(\varphi)}{\sqrt{P(\varphi)}}\frac{1}{f-\varphi}d\varphi\right)\cdot d\tau,$$

where $\sqrt{P(f)} \equiv \sqrt{f(f-2M)}$ and we will follow that branch of this root function which is real and positive for $f \equiv \varphi < 2M$, while M and b are a negative and a positive constant respectively, which could be determined by $\varphi(C) - \varphi(B) = V_C$ and by some regularity requirements for our solution [9].

Considering the correspondence between the curvilinear obstacle from the physical plane and the segment BC of the axis $\psi = 0$, the problem is much more complicated because we know on that portion of the boundary only $\theta = arctg(y'(x))$ but we do not know $V(x)$. To overpass this shortcoming we could extend the structure of f from the jet line λ to the whole domain of the flow by accepting

$$\frac{\frac{df}{d\tau}}{\frac{dg^*}{d\tau}} = \frac{dz}{d\theta}$$

and hence, an explicit dependence between z and τ (or f) could be established for BC too [9].

Finally if we evaluate the total pressure of the stream on our curvilinear obstacle (parachute) of perimeter L, we get that [7], [8]

$$\int_B^C (p-p_0)ds = (p_s^0 - p_0)\frac{L}{2} - \rho a\frac{L}{2} - \rho g\int_0^{x_C} x\sqrt{1+y'(x)}dx - \frac{1}{2}\int_0^{x_C}\left|\frac{df}{dz}\right|^2 ds.$$

Concerning the last term, taking into account that

$$\frac{df}{dz}\cdot\frac{dz}{d\tau} = \frac{M}{2} - \frac{M}{2\tau^2} = \frac{dg^*}{d\tau}$$

and

$$|z'(\tau)| = \frac{\frac{M}{2}\left(1-\frac{1}{\tau^2}\right)}{\left|\frac{dg^*}{d\tau}\right|}$$

we are led to

$$\int_B^C \left(\frac{df}{dz}\right)^2 ds = \frac{M}{2}\int_0^1 \left|\frac{dg^*}{d\tau}\right|\left(1-\frac{1}{\tau^2}\right)d\tau.$$

Let's now consider the case of a nonhomogeneous parachute of mass density $\rho_p(x)$ a parachutist of weight mg being linked to its "current" contour $\widehat{M'OM}$. We accept again the symmetry of the parachute versus the ascendant wind stream (Ox axis). Then, denoting by $L(t)$ the length of the subarc $OP(t)$ of the parachute, we have $L(t) = \int_0^t Y(u)du$, where $Y(u) = \sqrt{1+y'^2(u)}$, $y = y(x)$ being the equation of the parachute contour $M'OM \subset C'OC$, we could write that the equilibrium between the weight of the parachute-parachutist system and the whole action of the exterior fluid flow would be given by

$$2g\int_0^x \rho_p(t)L(t)dt + mg = 2\int_0^M (p-p_0)ds \equiv 2\int_0^x (p-p_0)Y(u)du.$$

But according to the already established results we also have, for the above right member,

$$p - p_0 = p_s^0 - p_0 - \rho a - g\rho x - \rho\frac{V^2}{2},$$

so that

$$\int_0^M (p-p_0)ds = \int_0^x (p_s^0 - p_0)Y(u)du - \rho a\int_0^x Y(u)du -$$
$$- g\rho\int_0^x uY(u)du - \frac{\rho}{2}\int_0^x V^2 Y(u)du.$$

Assuming the derivative of both members of this "equilibrium" condition we get an integral equation of the function $Y(x)$ which allows the study of the existence of its solution, i.e., of a contour $y(x)$ of the parachute implying the indefinitely floating. Namely we obtain, to within an additional constant,

$$2g\rho_p(x)\int_0^x Y(t)dt = 2(p_s^0 - p_0 - \rho a - \rho g x)Y(x) - \rho\left|\frac{df}{dz}\right|^2 Y(x),$$

or, by using some of the previous results,

$$\int_0^x Y(t)dt + k = \frac{p_s^0 - p_0 - \rho a - \rho g x}{g\rho_p(x)}Y(x) - \frac{\rho M}{4g\rho_p(x)}\left(1 - \frac{1}{\xi^2(x)}\right)\cdot$$
$$\cdot\left|[\xi(x)+1]^\alpha [b-\xi(x)]^{1/2} + [\xi(x)+1]^{\alpha+\varepsilon}\right|\cdot$$
$$\cdot\exp\left\{\frac{1}{\pi}\sqrt{\frac{1+\xi^2(x)}{2\xi}\left[\frac{1+\xi^2(x)}{2\xi} - 2\right]}\cdot I\cdot\xi'(x)\right\}\cdot Y(x),$$

where

$$I = \int_b^1 \frac{-\sqrt{Y^2(x(\xi))-1}}{\sqrt{\frac{(1+\xi^2)}{2\xi}\left(\frac{(1+\xi^2)}{2\xi} - 2\right)}}\cdot\frac{d\xi}{\frac{M}{2\xi}(1+\xi^2) - \xi},$$

$x = x(\xi)$ and its inverse $\xi = \xi(x)$ being "a priori" unknown sufficiently smooth functions ($x(\xi) \equiv x(\xi,\eta)|_{\eta=0} = \operatorname{Re}\tau(z)|_{\eta=0}$) while k is an additive constant chosen such that $Y(0)$ takes a certain value according to the value of $y'(0)$.

A single analysis shows that $\tau(0) = b$ while $\tau'(x)|_{CO} > 0$ and

$$\frac{df}{d\tau}|_{\tau\equiv\xi<1} = \frac{M}{2}\left(1 - \frac{1}{\xi^2}\right) > 0.$$

By choosing the branch $-\sqrt{Y^2(x)-1}$ for $y'(x)$ we have $y'(x) < 0$ for $x \in (x_C, 0)$. Concerning the integral

$$\int_b^1 \frac{y'(x(\xi))}{\sqrt{\frac{(1+\xi^2)}{2\xi}\left(\frac{(1+\xi^2)}{2\xi}-2\right)}} \cdot \frac{d\xi}{\frac{M}{2\xi}(1+\xi^2)-\xi},$$

it corresponds to the integral

$$\int_{\frac{(1+b)^2 M}{2b}}^{0} \frac{\mu(\varphi)}{\sqrt{P(\varphi)}} \frac{d\varphi}{f-\varphi},$$

which implies essentially only the integral

$$\int_{\frac{(1+b)^2 M}{2b}}^{2M} \frac{y'(x(\varphi))}{\sqrt{P(\varphi)}} \frac{d\varphi}{f-\varphi}$$

as $\mu(\varphi)$ is a known function on $[2M, 0]$.

In what follows we consider another form of this Volterra-Fredholm equation which seems to be more appropriate for a fixed-point analysis. More precisely, taking into account that $\frac{dg}{g\tau}|_{BC} = e^{y'(x(\xi))}|_{\xi\in(b,1)}$ we could replace $V^2Y(x)$ by $\frac{M}{2}\left|\frac{dg^*}{g\tau}\right|(1-\frac{1}{\tau^2})|_{\tau\equiv\xi\in(b,1)} \cdot Y(x)$, i.e.,

$$V^2Y(x) = \frac{M}{2}\left(1-\frac{1}{\xi^2(x)}\right)\left|[\xi(x)+1]^\alpha [b-\xi(x)]^{1/2} + [\xi(x)+1]^{\alpha+\varepsilon}\right| \cdot e^{y'(x(\xi))} \cdot Y(x),$$

where $e^{y'(x(\xi))} \approx e^{-Y(x)+\frac{1}{2Y(x)}}$.

Finally the "parachute parachutist equilibrium" equation in the unknown function $Y(x)$ is

$$\begin{aligned} \int_0^x Y(t)dt + k = {} & \frac{p_s^0 - p_0 - \rho a - \rho g x}{g\rho_p(x)} Y(x) - \\ & - \frac{M\rho}{4g\rho_p(x)}\left|[\xi(x)+1]^\alpha [b-\xi(x)]^{1/2} + [\xi(x)+1]^{\alpha+\varepsilon}\right| \cdot \\ & \cdot \left[1-\frac{1}{\xi^2(x)}\right] e^{-Y(x)+\frac{1}{2Y(x)}} \cdot Y(x) \end{aligned} \tag{2.1}$$

or, by denoting with

$$B(x) = \frac{p_s^0 - p_0 - \rho a - \rho g x}{g\rho_p(x)},$$

$$C(\xi(x)) = \frac{M\rho}{4g\rho_p(x)}\left|[\xi(x)+1]^\alpha [b-\xi(x)]^{1/2} + [\xi(x)+1]^{\alpha+\varepsilon}\right| \cdot \left[1-\frac{1}{\xi^2(x)}\right]$$

and

$$k = \left[B(0) - C(\xi(0))e^{-Y(0)+\frac{1}{2Y(0)}}\right]$$

we get the following "fixed point equation"

$$Y(x) = \frac{B(0) - C(\xi(0))e^{-Y(0)+\frac{1}{2Y(0)}}}{B(x) - C(\xi(x))e^{-Y(x)+\frac{1}{2Y(x)}}} + \int_0^x \frac{Y(t)}{B(x) - C(\xi(x))e^{-Y(x)+\frac{1}{2Y(x)}}} dt, \tag{2.2}$$

where we also know that $\alpha > 0$, $\varepsilon > 0$, $\alpha + \varepsilon < 1$, $p_s^0 - p_0 - \rho a > 0$, $y'(x(\xi)) \leq 0$ for $x_C < x \leq 0$ $(b \leq \xi < 1)$, $\xi(0) = b$ and consequently $B(x) > 0$ while $C(\xi(x)) > 0$.

3. Existence results for the indefinitely floating parachute

The equation (2.2) has the form

$$Y(x) = g\left(x, Y(x), Y(0)\right) + \int_0^x f\left(x, Y(x), Y(t)\right) dt,\ x \in \left[x_C, 0\right], \tag{3.1}$$

which is similar to that one considered in [10], [11].

Let us now consider a Banach space $X = \left(C\left([x_C, 0]\,; \mathbb{R}\right), ||\cdot||_B\right)$, where $||\cdot||_B$ is the Bielecki norm

$$||y||_B = \max_{\xi \in [x_C, 0]} |y(\xi)|\, e^{\tau \xi}$$

and the operator $A : X \to X$ defined through

$$A(Y)(x) = \frac{B(0) - C(\xi(0))e^{-Y(0)+\frac{1}{2Y(0)}}}{B(x) - C(\xi(x))e^{-Y(x)+\frac{1}{2Y(x)}}} + \int_0^x \frac{Y(t)}{B(x) - C(\xi(x))e^{-Y(x)+\frac{1}{2Y(x)}}} dt. \tag{3.2}$$

The solution of equation (2) satisfies $Y(0) = 1$ and, taking into account the physical conditions as well, let us define the following set

$$\mathcal{Y} = \left\{Y \in X | Y(0) = 1,\ 0 < Y_{\min} \leq Y(x) \leq Y_{\max},\ \forall x \in [x_C, 0]\right\}.$$

$\mathcal{Y}$ being a closed subset of X, it is a complete metric space and should be invariant versus the operator A.

Let us suppose that for any $x \in [x_C, 0]$,

$$\begin{aligned} &0 < B_{\min} \leq B(x) \leq B_{\max}, \\ &0 < C_{\min} \leq C(\xi(x)) \leq C_{\max}, \\ &Y \in \mathcal{Y}. \end{aligned} \tag{3.3}$$

So we have

$$e^{-Y_{\max}+\frac{1}{2Y_{\max}}} \leq e^{-Y(x)+\frac{1}{2Y(x)}} \leq e^{-Y_{\min}+\frac{1}{2Y_{\min}}}$$

and we can also suppose that

$$B_{\min} - C_{\max} e^{-Y_{\min}+\frac{1}{2Y_{\min}}} > 0. \tag{3.4}$$

It results immediately

$$\frac{k + Y_{\max} x_C}{B_{\max} - C_{\min} e^{-Y_{\max}+\frac{1}{2Y_{\max}}}} \leq A(Y)(x) \leq \frac{k}{B_{\min} - C_{\max} e^{-Y_{\min}+\frac{1}{2Y_{\min}}}}.$$

Accepting the conditions

$$\frac{k}{B_{\min} - C_{\max} e^{-Y_{\min}+\frac{1}{2Y_{\min}}}} \leq Y_{\max},\ \frac{k + Y_{\max} x_C}{B_{\max} - C_{\min} e^{-Y_{\max}+\frac{1}{2Y_{\max}}}} \geq Y_{\min}, \tag{3.5}$$

are simultaneously verified, one assures that $\mathcal{Y}$ is an invariant set versus A.

We intend now to apply a fixed point theory, namely to verify that A is a contraction on $\mathcal{Y}$.

First, let us denote

$$g(x,u)=\frac{k}{B(x)-C(\xi(x))e^{-u+\frac{1}{2u}}},\ u\in[Y_{\min},Y_{\max}]$$

and let's prove that

$$|g(x,u_1)-g(x,u_2)|\le L_g\,|u_1-u_2|\,,\ \forall u_1,u_2\in[Y_{\min},Y_{\max}]\,.$$

Indeed, we have

$$\left|\frac{\partial g}{\partial u}\right|=\frac{kC(\xi(x))e^{-u+\frac{1}{2u}}\cdot\left(1+\frac{1}{2u^2}\right)}{\left(B(x)-C(\xi(x))e^{-u+\frac{1}{2u}}\right)^2}$$

and from where

$$\max_{u\in[Y_{\min},Y_{\max}]}\left|\frac{\partial g}{\partial u}\right|\le\frac{kC_{\max}e^{-Y_{\min}+\frac{1}{2Y_{\min}}}\cdot\left(1+\frac{1}{2Y_{\min}^2}\right)}{\left(B_{\min}-C_{\max}e^{-Y_{\min}+\frac{1}{2Y_{\min}}}\right)^2}\equiv L_g. \tag{3.6}$$

Further, let introduce

$$f(x,u,v)=\frac{v}{B(x)-C(\xi(x))e^{-u+\frac{1}{2u}}},\ u,v\in[Y_{\min},Y_{\max}]$$

and let us show that

$$|f(x,u_1,v_1)-f(x,u_2,v_2)|\le L_{f,u}\,|u_1-u_2|+L_{f,v}\,|v_1-v_2|\,,$$

$\forall u_1,u_2,v_1,v_2\in[Y_{\min},Y_{\max}]\,.$

Indeed, we have

$$\left|\frac{\partial f}{\partial u}\right|=\left|\frac{vC(\xi(x))e^{-u+\frac{1}{2u}}\cdot\left(1+\frac{1}{2u^2}\right)}{\left(B(x)-C(\xi(x))e^{-u+\frac{1}{2u}}\right)^2}\right|,\ \left|\frac{\partial f}{\partial v}\right|=\left|\frac{1}{B(x)-C(\xi(x))e^{-u+\frac{1}{2u}}}\right|$$

from where

$$\max_{u,v\in[Y_{\min},Y_{\max}]}\left|\frac{\partial f}{\partial u}\right|\le\frac{Y_{\max}C_{\max}e^{-Y_{\min}+\frac{1}{2Y_{\min}}}\cdot\left(1+\frac{1}{2Y_{\min}^2}\right)}{\left(B_{\min}-C_{\max}e^{-Y_{\min}+\frac{1}{2Y_{\min}}}\right)^2}\equiv L_{f,u} \tag{3.7}$$

and

$$\max_{u,v\in[Y_{\min},Y_{\max}]}\left|\frac{\partial f}{\partial v}\right|\le\frac{1}{B_{\min}-C_{\max}e^{-Y_{\min}+\frac{1}{2Y_{\min}}}}\equiv L_{f,v}. \tag{3.8}$$

We also suppose

$$L_g-L_{f,u}x_C<1. \tag{3.9}$$

Theorem 3.1. *If conditions (3.4), (3.5), (3.9) are satisfied, then the equation (2.2) has a unique solution* $y^*\in\mathcal{Y}$.

Proof. Let us consider $A : \mathcal{Y} \to \mathcal{Y}$. We have the estimations

$$|A(y_1)(x) - A(y_2)(x)| \leq |g(x, y_1) - g(x, y_2)| +$$

$$+ \left| \int_0^x [f(x, y_1(x), y_1(t)) - f(x, y_2(x), y_2(t))] \, dt \right| \leq L_g |y_1(x) - y_2(x)| +$$

$$+ \int_x^0 (L_{f,u} |y_1(x) - y_2(x)| + L_{f,v} |y_1(t) - y_2(t)|) \, dt \leq L_g |y_1(x) - y_2(x)| -$$

$$-L_{f,u} x_C |y_1(x) - y_2(x)| \ + \frac{L_{f,v}}{\tau} ||y_1 - y_2||_B \, e^{-\tau x}$$

which implies

$$||A(y_1) - A(y_2)||_B \leq \left(L_g - L_{f,u} x_C + \frac{L_{f,v}}{\tau} \right) ||y_1 - y_2||_B$$

$\forall y_1, y_2 \in \mathcal{Y}$. The conclusion results now for a sufficiently large τ from the classical principle of contractions. □

So we can state that, under some imposed physical hypotheses, there could exist an improper shape (for its destination) of the parachute, namely a parachute which would "float indefinitely". Or in the language of the eolian turbines in an ascendant wind flow, there exists a shape of the turbine cup (bucket) which would stand, unable to be trained by the ascendant air flow.

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Received: March 24, 2011; Accepted: June 7, 2011.