SOME RELATIONSHIPS BETWEEN SUFFICIENT CONDITIONS FOR THE FIXED POINT PROPERTY

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Abstract. It is shown that E-convex Banach spaces satisfy the so called Prus-Sczepanik condition which in turn is sufficient for the fixed point property of nonexpansive mappings in Banach spaces. Moreover, we study the independence between several of these sufficient conditions.

Key Words and Phrases: Nonexpansive mappings, normal structure, fixed point property.

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