

SOME RENORMINGS WITH THE STABLE FIXED POINT PROPERTY

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Dedicated to K. Goebel, on the occasion of his retirement, and to L. Ćirić, W.A. Kirk and I.A. Rus on the occasion of their 75th birthday.

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Abstract. In this paper, we prove that for any number $\lambda < (\sqrt{33} - 3)/2$, any separable space X can be renormed in such a way that X satisfies the weak fixed point property for non-expansive mappings and this property is inherited for any other isomorphic space Y such that the Banach-Mazur distance between X and Y is less than λ . We also prove that any, in general nonseparable, Banach space with an extended unconditional basis can be renormed to satisfy the w-FPP with the same stability constant.

Key Words and Phrases: fixed point, non-expansive mapping, Banach-Mazur distance, fixed point property.

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