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STRONG CONVERGENCE OF GENERAL MODIFIED MANN ITERATIONS FOR STRICT PSEUDO-CONTRACTIONS IN HILBERT SPACES

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Abstract. In this paper, we use a general iterative method that contains algorithms defined by G.Marino, H.K.Xu and Yamada to modify the normal Mann's iterative process to have strong convergence for a $k-$strictly pseudo-contractive mapping in the framework of Hilbert spaces. Our results improve and extend the corresponding results announced by many others.
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1. Introduction

Let H be a real Hilbert space and K be a nonempty closed convex subset of H. Recall that a mapping $T : K \to H$ on H which is said to be k-strictly pseudo-contractive if there exists a constant $k \in [0,1)$ such that

$$\|Tx - Ty\|^2 \leq \|x-y\|^2 + k\|(I-T)x - (I-T)y\|^2, \text{ for all } x, y \in K. \tag{1.1}$$

Note that the class of k-strict pseudo-contraction strictly includes the calss of nonexpansive mapping T on K such that .

$$\|Tx - Ty\|^2 \leq \|x-y\|^2, \text{ for all } x, y \in K. \tag{1.2}$$

That is, T is nonexpansive if and only if T is 0-srictly pseudo-contractive,it is also said to be pseudo-contractive if $k = 1$.

T is said to be strongly pseudo-contractive if there exists a positive constant $\lambda \in (0,1)$ such that $T + \lambda I$ is pseudo-contractive. It is obvious that the class of k-strict pseudo-contractions falls into the one between classes of nonexpansive mappings and pseudo-contractions.

But we should know that the class of strongly pseudo-contractive mappings is independent of the class of $k-$strict pseudo-contractions. It is easy to prove that , in a real Hilbert space H, (1.1) is equivalent to

$$\langle Tx - Ty, x-y\rangle \leq \|x-y\|^2 - \frac{1-k}{2}\|(I-T)x - (I-T)y\|^2, \text{ for all } x, y \in K. \tag{1.3}$$

T is pseudo-contractive if and only if

$$\langle Tx - Ty, x - y\rangle \leq \|x - y\|^2, \text{ for all } x, y \in K. \tag{1.4}$$

T is strongly pseudo-contractive if and only if there exists a positive constant $\lambda \in (0, 1)$ such that

$$\langle Tx - Ty, x - y\rangle \leq (1 - \lambda)\|x - y\|^2, \text{ for all } x, y \in K. \tag{1.5}$$

In 1953, Mann [4] introduced the normal Mann's iterative process as follows

$$\forall x_1 \in K, x_{n+1} = (1 - \alpha_n)x_n + \alpha_n Tx_n, \forall n \geq 1, \tag{1.6}$$

where the sequences $\{\alpha_n\}_{n=0}^{\infty}$ is in the interval (0,1). If T is nonexpansive mapping with a fixed point and the sequence $\{\alpha_n\}$ is chosen so that $\sum_{n=0}^{\infty} \alpha_n(1 - \alpha_n) = \infty$, then the sequence $\{x_n\}$ generated by normal Mann's iterative process (1.6) converges weakly to a fixed point of T. In 1967, Browder and Petryshyn [1] established the first convergence result for k-strictly pseudo-contractive self-mappings in real Hilbert spaces. They proved weak and strong convergence Theorems by using algorithm(1.6) with a constant control sequence $\{\alpha_n\} = \alpha$ for all n. Afterward, Rhoades [11] generalized in part the corresponding results in [2] in the sense that a variable control sequence $\{\alpha_n\}$ was taken into consideration. However, without the compact assumption on the domain of mapping T, in general, one cannot expect to infer any weak convergence results from Rhoades'convergence Theorem.

Lot of works have been done for the modification of the normal Mann's iteration so that strong convergence is guaranteed. See, e.g., [2, 5, 8, 9] and the reference therein.

Kim and Xu [3] introduced the following iteration process

$$\begin{cases} x_0 = x \in K \quad \text{arbitrarily chosen}, \\ y_n = \beta_n x_n + (1 - \beta_n)Tx_n, \\ x_{n+1} = \alpha_n u + (1 - \alpha_n)y_n, \forall n \in N, \end{cases} \tag{1.7}$$

where T is a nonexpansive mapping of K into itself, $u \in K$ is a given point. They proved the sequence $\{x_n\}$ defined by (1.7) convergences strongly to a fixed point of T provided the sequences $\{\alpha_n\}$ and $\{\beta_n\}$ satisfy appropriate conditions.

Yao et al. [14] also modified Mann's iterative scheme (1.6) by using so-called viscosity approximation method which was introduced by Moudafi [7]

$$\begin{cases} x_0 = x \in K \quad \text{arbitrarily chosen}, \\ y_n = \beta_n x_n + (1 - \beta_n)Tx_n, \\ x_{n+1} = \alpha_n f(x_n) + (1 - \alpha_n)y_n, \forall n \in N, \end{cases} \tag{1.8}$$

where T is nonexpansive mapping of K into itself and f is a contraction on K. They obtained a strong convergence theorem under some mild restrictions on parameters. Recently, Marino and Xu [6] introduced the following iterative algorithm

$$x_0 = x \in H, x_{n+1} = \alpha_n \gamma f(x_n) + (I - \alpha_n A)Tx_n, n \geq 0, \tag{1.9}$$

where T is a self-nonexpansive mapping on H, A is a strong positive bounded linear operator on H. They proved the sequence defined by above iterative process converges strongly to a fixed point of T which is a unique solution of the variation inequality

$$\langle(\gamma f - A)x^*, x - x^*\rangle \leq 0,$$

for all $x \in F(T)$, and is also the optimality condition for some minimization problem.

Xiaolong Qin et al. [10] introduced a composite iteration scheme as follows:

$$\begin{cases} x_0 = x \in K \quad \text{arbitrarily} \quad \text{chosen}, \\ y_n = P_K(\beta_n x_n + (1-\beta_n)Tx_n), \\ x_{n+1} = \alpha_n \gamma f(x_n) + (I - \alpha_n A)y_n, \forall n \in N, \end{cases} \tag{1.10}$$

where T is non-self $k-$strict pseudo-contraction, f is a contraction and A is strong positive bounded linear operator on H. They proved that $\{x_n\}$ defined by (1.10) converges strongly to a fixed point of the $k-$strict pseudo-contraction which solves some variation inequality.

Very recently, M.Tian [12] proposed a general iterative method for Nonexpansive Mappings that contains algorithms defined by G. Marino, H.K. Xu and Yamada:

$$x_0 = x \in H, x_{n+1} = \alpha_n \gamma f(x_n) + (I - \mu \alpha_n F)Tx_n, n \geq 0, \tag{1.11}$$

where T is a self-nonexpansive mapping on H, F is $L-$Lipschitzian continuous and $\eta-$strongly monotone operator on H and f is a contraction on H. Then M.Tian[13] proved the sequence defined by above iterative process converges strongly to a fixed point of T.

In this paper, inspired and motivated by the above works, we use a more general iterative scheme to modify the Mann's iterative process

$$\begin{cases} x_0 = x \in H \quad \text{arbitrarily} \quad \text{chosen}, \\ y_n = \beta_n x_n + (1-\beta_n)Tx_n, \\ x_{n+1} = \alpha_n \gamma f(x_n) + (1 - \mu \alpha_n F)y_n, \forall n \in N, \end{cases} \tag{1.12}$$

where $T : H \to H$ is a $k-$strict pseudo-contraction , f is a contraction on H and F is $L-$Lipschitzian continuous and $\eta-$strongly monotone operator on H. Under certain appropriate assumptions on the sequences $\{\alpha_n\}$ and $\{\beta_n\}$, we prove that $\{x_n\}$ generated by (1.12) converges strongly to a fixed point of the $k-$strict pseudo-contraction which solves some variation inequality. In order to prove our main results , we need the following definitions and Lemmas.

Throughout this paper , we use $F(T)$ to denote the fixed point set of the mapping T and P_K to denote the metric projection of H onto its closed convex subset K. Recall that a self mapping $f : H \to H$ is a contraction if there exists a constant $\alpha \in (0,1)$ such that

$$\|f(x) - f(y)\| \leq \alpha \|x - y\|, \quad \text{for} \quad \text{all} \quad x, y \in H.$$

We use Π_H to denote the collection of all contractions on H. That is

$$\Pi_H = \{f | f : H \to H \quad \text{a} \quad \text{contraction}\}.$$

F is a $L-$Lipschitzian continuous and $\eta-$strongly monotone operator with $L, \eta > 0$.

2. Main results

Lemma 2.1. ([15]) If T is a $k-$strict pseudo-contraction on a closed convex subset of K of a Hilbert space H, then the fixed point set $F(T)$ is closed convex so that the projection $P_{F(T)}$ is well defined.

Lemma 2.2. ([1]) Let $T : K \to H$ be a k-strict pseudo-contraction. Define $S : K \to H$ by $Sx = \lambda x + (1 - \lambda)Tx$ for each $x \in K$. Then, as $\lambda \in [k, 1)$, S is nonexpansive mapping such that $F(S) = F(T)$.

Lemma 2.3. In a Hilbert space H, there holds the inequality:

$$\|x + y\|^2 \le \|x\|^2 + 2\langle y, (x + y)\rangle, \forall x, y \in H.$$

Lemma 2.4. ([13]) Assume that $\{\alpha_n\}$ is a sequence of nonnegative real numbers such that

$$\alpha_{n+1} \le (1 - \gamma_n)a_n + \gamma_n\delta_n, n \ge 0,$$

where $\{\gamma_n\}$ is a sequence in (0,1) and $\{\delta_n\}$ is a sequence in $\mathbb{R}$ such that

(i) $\sum_{n=1}^{\infty} \gamma_n = \infty$;

(ii) $\limsup_{n\to\infty} \delta_n \le 0 \quad or \quad \sum_{n=1}^{\infty} |\delta_n\gamma_n| < \infty$.

Then $\lim_{n\to\infty} \alpha_n = 0$.

Lemma 2.5. ([5]) Let H be a real Hilbert space, there holds the following identities:

(i) $\|x \pm y\|^2 = \|x\|^2 \pm 2\langle x, y\rangle + \|y\|^2, \forall x, y \in H$;

(ii) $\|tx + (1 - t)y\|^2 = t\|x\|^2 + (1 - t)\|y\|^2 - t(1 - t)\|x - y\|^2, \forall t \in [0, 1], \forall x, y \in H$.

Lemma 2.6. Let F be a $L-$Lipschitzian continuous operator and $\eta - strongly$ monotone operator on a Hilbert space H with $L, \eta > 0$. Then for $0 < \mu < 2\eta/L^2$, and $0 < t < 1$. Then $S = I - t\mu F : H \to H$ is a contraction with contractive coefficient $1 - t\tau$ and $\tau = \frac{1}{2}\mu(2\eta - \mu L^2)$.

Proof. $\|Sx - Sy\|^2 = \|x - y - t\mu(Fx - Fy)\|^2$

$$= \|x - y\|^2 + t^2\mu^2\|Fx - Fy\|^2 - 2t\mu\langle Fx - Fy, x - y\rangle$$

$$\le \|x - y\|^2 + t^2\mu^2L^2\|x - y\|^2 - 2t\mu\eta\|x - y\|^2$$

$$\le [1 - t\mu(2\eta - \mu L^2)]\|x - y\|^2 \le (1 - t\tau)^2\|x - y\|^2,$$

where $\tau = \frac{1}{2}\mu(2\eta - \mu L^2)$, and

$$\|Sx - Sy\| \le (1 - t\tau)\|x - y\|,$$

hence S is a contraction with contractive coefficient $(1 - t\tau)$.

Theorem 2.7. Let H be real Hilbert space, $f \in \Pi_H$ with the coefficient $\alpha \in (0, 1)$, let F be a $L-Lipschitzian$ continuous and $\eta-strongly$ monotone operator on H with $L, \eta > 0$. Assume that $0 < \mu < \frac{2\eta}{L^2}$, $0 < \gamma < \frac{\mu(\eta - \frac{\mu L^2}{2})}{\alpha} = \frac{\tau}{\alpha}$ and let $T : H \to H$ be a k-strictly pseudo-contractive mapping such that $F(T) \neq \phi$. Given sequence $\{\alpha_n\}_{n=0}^{\infty}$ and $\{\beta_n\}_{n=0}^{\infty}$ in [0, 1] , the following control conditions are satisfied

(i) $\{\alpha_n\} \subset (0, 1)$, $\lim_{n\to\infty} \alpha_n = 0, \sum_{n=1}^{\infty} \alpha_n = \infty$,

(ii) $0 \le k \le \beta_n \le \lambda < 1$, for all $n \ge 1$, $\lim_{n\to\infty} \beta_n = \lambda$,

(iii) $\sum_{n=1}^{\infty} |\alpha_{n+1} - \alpha_n| < \infty \quad and \quad \sum_{n=1}^{\infty} |\beta_{n+1} - \beta_n| < \infty$,

let $\{x_n\}_{n=1}^{\infty}$ be the sequence generated by the composite process(1.12). Then $\{x_n\}_{n=1}^{\infty}$ converges strongly to $q \in F(T)$ which also solve the following variational inequality

$$\langle(\mu F - \gamma f)q, p - q\rangle \ge 0, \quad for \quad all \quad p \in F(T). \tag{2.1}$$

Proof. We divide the proof into three parts.

Step 1. First, we show that sequence $\{x_n\}_{n=0}^{\infty}$ and $\{y_n\}_{n=0}^{\infty}$ are bounded, taking a point $p \in F(T)$, we obtain

$$\begin{aligned}\|y_n - p\|^2 &= \|\beta_n x_n + (1-\beta_n)Tx_n - p\|^2 \\ &= \|\beta_n(x_n - p) + (1-\beta_n)(Tx_n - p)\|^2 \\ &= \beta_n\|(x_n - p)\|^2 + (1-\beta_n)\|(Tx_n - p)\|^2 - \beta_n(1-\beta_n)\|(Tx_n - x_n)\|^2 \\ &\le \|(x_n - p)\|^2 - (\beta_n - k)(1-\beta_n)\|(Tx_n - x_n)\|^2 \\ &\le \|x_n - p\|^2.\end{aligned}$$

It follows that

$$\begin{aligned}\|x_{n+1} - p\| &= \|\alpha_n(\gamma f(x_n) - \mu Fp) + (I - \mu\alpha_n F)y_n - (I - \mu\alpha_n F)p\| \\ &\le \alpha_n\gamma\alpha\|x_n - p\| + \alpha_n\|\gamma f(p) - \mu F(p)\| + (1 - \alpha_n\tau)\|x_n - p\| \\ &\le (1 - \alpha_n(\tau - \gamma\alpha))\|x_n - p\| + \alpha_n\|\gamma f(p) - \mu F(p)\|.\end{aligned}$$

By induction, we have

$$\|x_n - p\| \le \max\{\|x_1 - p\|, \frac{1}{\tau - \gamma\alpha}\|\gamma f(p) - \mu F(p)\|\}, \forall n \in N,$$

which gives that the sequence $\{x_n\}$ is bounded, so is $\{y_n\}$.

Step 2. We shall claim that $\|x_{n+1} - x_n\| \to 0$, as $n \to \infty$. Define mapping $T_n x = \beta_n x + (1-\beta_n)Tx$ for each $x \in H$. Then $T_n : H \to H$ is nonexpansive , indeed, by using (1. 1) and Lemma 2.5, we have for all $x, y \in H$

$$\begin{aligned}\|T_n x - T_n y\|^2 &= \|(\beta_n I + (1-\beta_n)T)x - (\beta_n I + (1-\beta_n)T)y\|^2 \\ &= \beta_n\|x - y\|^2 + (1-\beta_n)\|Tx - Ty\|^2 - \beta_n(1-\beta_n)\|(I-T)x - (I-T)y\|^2 \\ &\le \beta_n\|x - y\|^2 + (1-\beta_n)[\|x - y\|^2 + k\|(I-T)x - (I-T)y\|^2] \\ &\quad -\beta_n(1-\beta_n)\|(I-T)x - (I-T)y\|^2 \\ &\le \|x - y\|^2,\end{aligned}$$

which implies that $T_n : H \to H$ is nonexpansive, therefore by

$$x_{n+1} = \alpha_n\gamma f(x_n) + (1 - \mu\alpha_n F)T_n x_n, \tag{2.2}$$

it follows that

$$\begin{aligned}x_{n+2} - x_{n+1} &= (I - \alpha_{n+1}\mu F)T_{n+1}x_{n+1} - (I - \alpha_{n+1}\mu F)T_n x_n \\ &\quad -(\alpha_{n+1} - \alpha_n)\mu F T_n x_n + \gamma[\alpha_{n+1}(f(x_{n+1}) - f(x_n)) \\ &\quad +f(x_n)(\alpha_{n+1} - \alpha_n)],\end{aligned} \tag{2.3}$$

which yields that

$$\begin{aligned}\|x_{n+2} - x_{n+1}\| &\le (1 - \alpha_{n+1}\tau)(\|x_{n+1} - x_n\| + \|T_{n+1}x_n - T_n x_n\|) \\ &\quad +|\alpha_{n+1} - \alpha_n|\mu\|FT_n x_n\| + \gamma[\alpha_{n+1}\alpha\|x_{n+1} - x_n\| \\ &\quad +\|f(x_n)\||\alpha_{n+1} - \alpha_n|].\end{aligned}$$

Notice that

$$\begin{aligned}\|T_{n+1}x_n - T_n x_n\| &= \|[\beta_{n+1}x_n + (1-\beta_{n+1})Tx_n] - [\beta_n x_n + (1-\beta_n)Tx_n]\| \\ &= \|x_n - T_n x_n\||\beta_{n+1} - \beta_n|,\end{aligned} \tag{2.4}$$

and so

$$\|x_{n+2}-x_{n+1}\| \leq [1-\alpha_{n+1}(\tau-\gamma\alpha)]\|x_{n+1}-x_n\|+M_1(|\alpha_{n+1}-\alpha_n|+|\beta_{n+1}-\beta_n|)], \quad (2.5)$$

where

$$M_1 \geq \|x_n - T_n x_n\| + \gamma\|f(x_n)\| + \mu\|FT_n x_n\|, \quad \text{for all} \quad n.$$

By applying Lemma 2.4, we have

$$\lim_{n\to\infty} \|x_{n+1} - x_n\| = 0. \quad (2.6)$$

Step 3. Finally , we claim $x_n \to q$ as $n \to \infty$. It is obvious that

$$\|T_n x_n - x_n\| \leq \|x_n - x_{n+1}\| + \alpha_n(\gamma\|f(x_n\| + \mu\|FT_n x_n\|),$$

so

$$\lim_{n\to\infty} \|T_n x_n - x_n\| = 0. \quad (2.7)$$

On the other hand, we have $\beta_n \to \lambda$ as $n \to \infty$, where $\lambda \in [k, 1)$. Define $S : K \to H$ by $Sx = \lambda x + (1-\lambda)Tx$, then, S is nonexpansive with $F(S) = F(T)$ by Lemma 2.2 . Notice that

$$\begin{aligned} \|Sx_n - x_n\| &\leq \|x_n - T_n x_n\| + \|T_n x_n - Sx_n\| \\ &\leq \|x_n - T_n x_n\| + \|\beta_n x_n + (1-\beta_n)Tx_n - [\lambda x_n + (1-\lambda)Tx_n]\| \\ &\leq \|x_n - T_n x_n\| + |\beta_n - \lambda|\|x_n - Tx_n\|, \end{aligned}$$

which combines with (2.7) yielding that

$$\lim_{n\to\infty} \|Sx_n - x_n\| = 0. \quad (2.8)$$

Now, we claim that

$$\limsup_{n\to\infty} \langle(\mu F - \gamma f)q, x_n - q\rangle \geq 0, \quad (2.9)$$

where q is the unique solution in $F(T)$ to the variational inequality

$$\langle(\mu F - \gamma f)q, p - q\rangle \geq 0, p \in F(T).$$

Since $\{x_n\}$ is bounded, with out loss of generality let $x_{n_k} \rightharpoonup \bar{x}$, by Lemma 2.2 and (2.8)

$$\bar{x} \in F(S), \text{ so } \bar{x} \in F(T).$$

$$\begin{aligned} \limsup_{n\to\infty} \langle(\mu F - \gamma f)q, x_n - q\rangle &= \lim_{k\to\infty} \langle(\mu F - \gamma f)q, x_{n_k} - q\rangle \\ &= \langle(\mu F - \gamma f)q, \bar{x} - q\rangle \geq 0. \quad (2.10) \end{aligned}$$

Finally, we prove that $x_n \to q$.

$$x_{n+1} = \alpha_n \gamma f(x_n) + (1 - \mu\alpha_n F)y_n$$

so,from

$$x_{n+1} - q = \alpha_n(\gamma f(x_n) - \mu Fq) + (I - \mu\alpha_n F)y_n - (I - \mu\alpha_n F)q,$$

we have

$$\begin{aligned} \|x_{n+1} - q\|^2 &\leq \|(I - \mu\alpha_n F)y_n - (I - \mu\alpha_n F)q\|^2 + 2\alpha_n\langle\gamma f(x_n) - \mu Fq, x_{n+1} - q\rangle \\ &\leq (1 - \alpha_n\tau)^2\|x_n - q\|^2 + 2\alpha_n\gamma\langle f(x_n) - f(q), x_{n+1} - q\rangle + 2\alpha_n\langle\gamma f(q) - \mu Fq, x_{n+1} - q\rangle \\ &\leq (1 - \alpha_n\tau)^2\|x_n - q\|^2 + 2\alpha_n\gamma\alpha\|x_n - q\|\|x_{n+1} - q\| + 2\alpha_n\langle\gamma f(q) - \mu Fq, x_{n+1} - q\rangle \end{aligned}$$

$$\leq ((1-\alpha_n\tau)^2+\alpha_n\gamma\alpha)\|x_n-q\|^2+\alpha_n\gamma\alpha\|x_{n+1}-q\|^2+2\alpha_n\langle\gamma f(q)-\mu Fq, x_{n+1}-q\rangle. \tag{2.11}$$

This implies that

$$\begin{aligned}
\|x_{n+1}-q\|^2 &\leq \frac{1-2\alpha_n\tau+(\alpha_n\tau)^2+\alpha_n\gamma\alpha}{1-\alpha_n\gamma\alpha}\|x_n-q\|^2\\
&\quad+\frac{2\alpha_n}{1-\alpha_n\gamma\alpha}\langle\gamma f(q)-\mu Fq, x_{n+1}-q\rangle\\
&=(1-\frac{2\alpha_n(\tau-\gamma\alpha)}{1-\alpha_n\gamma\alpha})\|x_n-q\|^2+\frac{(\alpha_n\tau)^2}{1-\alpha_n\gamma\alpha}\|x_n-q\|^2\\
&\quad+\frac{2\alpha_n}{1-\alpha_n\gamma\alpha}\langle\gamma f(q)-\mu Fq, x_{n+1}-q\rangle\\
&=(1-\frac{2\alpha_n(\tau-\gamma\alpha)}{1-\alpha_n\gamma\alpha})\|x_n-q\|^2+\frac{2\alpha_n(\tau-\gamma\alpha)}{1-\alpha_n\gamma\alpha}\\
&\quad\{\frac{\alpha_n\tau^2}{2(\tau-\gamma\alpha)}M^*+\frac{1}{\tau-\gamma\alpha}\langle\gamma f(q)-\mu Fq, x_{n+1}-q\rangle\}\\
&=(1-\gamma_n)\|x_n-q\|^2+\gamma_n\delta_n,
\end{aligned} \tag{2.12}$$

where

$$M^*=\sup\{\|x_n-q\|^2 : n\in N\},\ \gamma_n=\frac{2\alpha_n(\tau-\gamma\alpha)}{1-\alpha_n\gamma\alpha}$$

and

$$\delta_n=\frac{\alpha_n\tau^2}{2(\tau-\gamma\alpha)}M^*+\frac{1}{\tau-\gamma\alpha}\langle\gamma f(q)-\mu Fq, x_{n+1}-q\rangle.$$

It is easily to see that $\gamma_n\to 0, \sum_{n=1}^{\infty}\gamma_n=\infty$ and $\limsup_{n\to\infty}\delta_n\leq 0$ by (2.10). Hence by Lemma 2.4, the sequence $\{x_n\}$ converges strongly to q.

Remark 2.8. If $f=u, \gamma=1, \mu=1, F=I, k=0$, then Theorem 2.7 reduces to Theorem 1 of Kim and Xu [3].

Remark 2.9. If $\gamma=1, \mu=1, F=I, k=0$,then Theorem 2.7 reduces to Theorem 1 of Yao et al. [14].

Remark 2.10. If $\mu=1, F=A$, then Theorem 2.7 reduces to Theorem 2.1 of Xiaolong Qin et al. [10].

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